

1. Let $f(x) = -\ln(1 - x)$.

(a) Find $f'(x)$, $f''(x)$, $f'''(x)$, $f^{(4)}(x)$, and $f^{(n)}(x)$

(b) Find $f(0)$, $f'(0)$, $f''(0)$, $f'''(0)$, $f^{(4)}(0)$, and $f^{(n)}(0)$

(c) Find the n th-degree Maclaurin polynomial of f

(d) Find the Maclaurin series for $f(x)$. (Do not verify that $\lim_{n \rightarrow \infty} R_n(x) = 0$.)

(e) Find the interval of convergence for this Maclaurin series.

2. Find the first five terms of the Taylor series for the following functions at a given value c :

(a) $f(x) = \cos x$ at $c = \frac{\pi}{3}$

(b) $f(x) = \sqrt{x}$ at $c = 4$

(c) $f(x) = \tan x$ at $c = \frac{\pi}{4}$

3. Find the Maclaurin series for $f(x) = 6x^4 - 2x^3 + 4x^2 + x + 7$. Show all work!

4. Find the sum of each of the following infinite series. Give exact answers.

(a) $1 - \frac{1}{2!} + \frac{1}{4!} - \frac{1}{6!} + \cdots + (-1)^n \frac{1}{(2n)!} + \cdots$

(b) $1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \cdots + (-1)^n \frac{1}{n+1} + \cdots$

(c) $\frac{\pi}{6} - \frac{\pi^3}{6^3 \cdot 3!} + \frac{\pi^5}{6^5 \cdot 5!} - \cdots + (-1)^n \frac{\pi^{2n+1}}{6^{2n+1} (2n+1)!} + \cdots$

5. (a) Use the Maclaurin series for $\arctan x$ with $x = 1$ to represent π as the sum of an infinite series.
- (b) What accuracy is obtained by using the first five terms of the series to approximate π ?
- (c) Approximately how many terms of the series are required to obtain four decimal places of accuracy?