1. (Modified from the 2007 AP Calculus AB exam) Let R be the region in the first and second quadrants that is bounded above by the graph of $y = \dfrac{20}{1+x^2}$ and below by the horizontal line $y = 2$. Let S be the region in the first quadrant bounded above by $y = \dfrac{20}{1+x^2}$ and below by the x-axis. (This problem continues on the following pages.)

   (a) Evaluate $\displaystyle\int \frac{1}{(1+x^2)^2}\,dx$

   (b) Evaluate $\displaystyle\int_0^3 \frac{1}{(1+x^2)^2}\,dx$.

   (c) Evaluate $\displaystyle\int_0^\infty \frac{1}{(1+x^2)^2}\,dx$.

   (d) Find the volume of the solid generated when R is rotated about the x-axis.

(e) Find the volume of the solid generated when R is rotated about the y-axis.

(f) Find the volume of the solid generated when S is rotated about the x-axis.

(g) Find the volume of the solid generated when S is rotated about the y-axis.

(h) Setup, but do not evaluate, an integral expression that gives the length of the boundary curve for the region R.

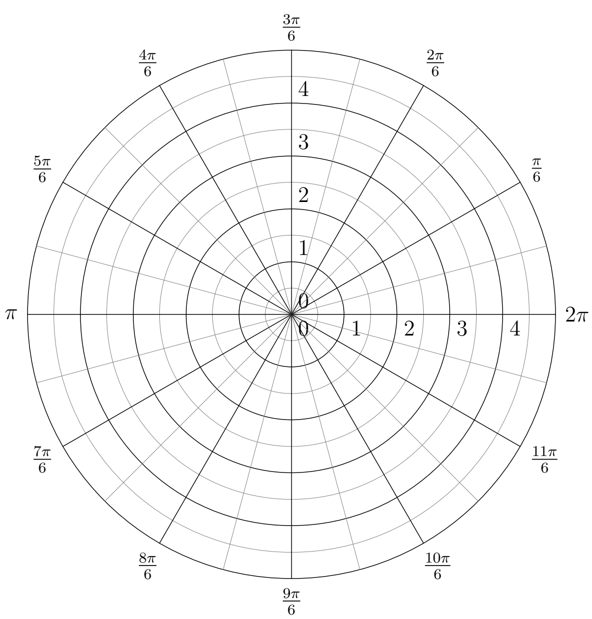
(i) Setup, but do not evaluate, an integral expression that gives the surface area of the solid that is formed when the region R is rotated about the line $y = 2$.

(j) The region R is the base of a solid. For this solid, the cross-sections perpendicular to the x-axis are semicircles. Find the volume of this solid.

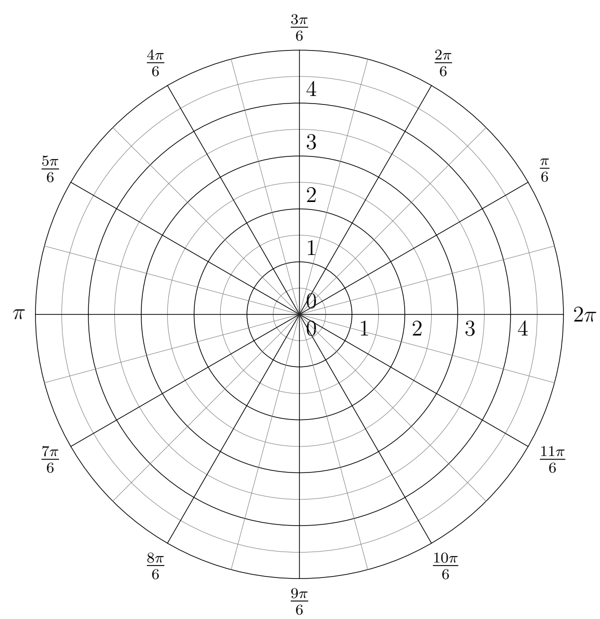
2. A person standing on a platform 20 ft above the ground is holding one end of a chain. On the other end of the chain is a 10 lb weight. If the chain weighs 2 lbs/ft. How much work is required to lift the weight to the top of the platform?

3. Graph the following equations in polar coordinates.

(a) $r = 2 - 2\cos\theta$



(b) $r = -3\sin\theta$



4. Find an equation of the tangent line to $r = 3\cos\theta$ at the points where $\theta = \pi/4$ and $\theta = \pi/2$.

5. Convert the following equations from polar to rectangular coordinates.

   (a) $r = 7\sin\theta$

   (b) $r^2 = \tan\theta$

6. Convert the following equations from rectangular to polar coordinates.

   (a) $y = x^2$

   (b) $(x+3)^2 + y^2 = 9$

7. Sketch and find the area of the region inside of the cardioid $r = 1 + \cos\theta$ and outside of the circle $r = 1$.

8. Compute the following integrals.

(a) $\int 5^x dx$

(b) $\int \frac{x}{1-x^2} dx$

(c) $\int xe^x dx$

(d) $\int \frac{x^3-5x^2+6x+1}{x^2-5x+6} dx$

(e) $\int \frac{1}{\sqrt{9-x^2}} dx$

(f) $\int \ln x dx$

(g) $\int \sin^3 x \cos^5 x dx$

(h) $\int \frac{x^2}{(1-x^2)^{3/2}}$

9. Determine whether the following sequences converge or diverge. For those that converge, find the limit.

   (a) $\left\{1 + \dfrac{(-1)^n}{n}\right\}$

   (b) $\left\{\dfrac{4^n - 7}{9^n}\right\}$

10. Find the sum of each series:

    (a) $\displaystyle\sum_{n=2}^{\infty} e^{-n}$

    (b) $\displaystyle\sum_{n=1}^{\infty} \frac{2}{(2n-3)(2n-1)}$

11. Determine whether the following series converge or diverge. Make sure to show all work leading to your conclusion.

    (a) $\displaystyle\sum_{n=1}^{\infty} \frac{3n+5}{n^2+7}$

    (b) $\displaystyle\sum_{n=3}^{\infty} \frac{1}{n\sqrt{\ln n}}$

(c) $\displaystyle\sum_{n=1}^{\infty} \frac{2^n 3^n}{n^n}$

12. Determine whether the following series are absolutely convergent, conditionally convergent, or divergent. Make sure to show all work leading to your conclusion.

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{n\sqrt{n^2+1}}$

(b) $\displaystyle\sum_{n=1}^{\infty} (-1)^n \frac{\ln n}{3+\ln n}$

(c) $\displaystyle\sum_{n=1}^{\infty} (-1)^n \frac{3n^2}{n^3+1}$

13. For each of the following power series, find the interval of convergence and/or the radius of convergence:

(a) $\displaystyle\sum_{n=0}^{\infty} \frac{1}{n3^n}(x+4)^n$

Interval of Convergence: ____________

Radius of Convergence: ____________

(b) $\displaystyle\sum_{n=0}^{\infty} \frac{n!}{n^n} x^n$

Radius of Convergence: ____________

14. Find a power series in x that has the given function as its sum. Also find the interval of convergence.

(a) $\dfrac{1}{1+x^3}$

(b) $\sin \dfrac{2x}{3}$

15. (a) Find the 4^{th}-degree Taylor Polynomial for the function $f(x) = \dfrac{1}{1-x}$ centered at $c = 2$, and find the Taylor remainder for $n = 4$.

(b) Use the Taylor Polynomial to approximate $\frac{1}{1-2.1}$.

(c) Use the Taylor Remainder to determine the error in your approximation.

16. Find the first four terms of the Binomial series $(1 - x^2)^{1/2}$.

17. Approximate the integral $\displaystyle\int_0^1 x\sin(x^3)\,dx$ to six decimal places.