

1. The base of a solid is the region bounded by $y = x^2$ and $y = 2 - x^2$. Find the volume of the solid if every cross section by a plane perpendicular to the x -axis is:

(a) a square

(b) a semicircle

(c) an equilateral triangle

2. The base of a solid is the region bounded by $x = \sqrt{5}y^2$, $y = 2$, and the y -axis.

(a) Find the volume of the solid if every cross section by a plane perpendicular to the y -axis is a semicircle.

(b) Find the volume of the solid if every cross section by a plane perpendicular to the x -axis is a semicircle.

3. Volume of a Wedge

A plane inclined at an angle of 45° passes through a diameter of the base of a right circular cylinder of radius r . Find the volume of the region within the cylinder and below the plane. (This volume was first calculated in the third century BCE by Archimedes, who also derived the formula $V = \frac{4}{3}\pi r^3$ for the volume of a sphere of radius r . His work on the wedge is found in a manuscript that was discovered in 1906 after having been lost for centuries.)

4. A frustum of a right pyramid is a right pyramid with its top cut off. Let V be the volume of a frustum of height h whose base is a square of side a and top is a square of side b with $a > b \geq 0$. Show that $V = \frac{h}{3}(a^2 + ab + b^2)$. (A papyrus dating to the year 1850 BCE indicates that Egyptian mathematicians had discovered this formula almost 4,000 years ago.) [Hint: This problem can be done by modifying the method of example 1 on page 329 of your textbook. It can also be done without using calculus if you begin with the result of this example. Try both ways!]