1. Find each limit, (if it exists).

(a) $\lim\limits_{x\to 0+} \dfrac{\ln x}{\csc x}$

(b) $\lim\limits_{x\to 1} \dfrac{2x^2-(3x+1)\sqrt{x}+2}{x-1}$

(c) $\lim\limits_{x\to \infty} \dfrac{\sqrt{9x+1}}{\sqrt{x+1}}$

(d) $\lim\limits_{x\to \frac{\pi}{4}} \dfrac{\sin x-\cos x}{x-\frac{\pi}{4}}$

(e) $\lim\limits_{x\to 0} \dfrac{x\tan x}{1-\cos x}$

(f) $\lim\limits_{x\to \infty} \dfrac{2x^2-1}{5x^2+3x}$

(g) $\lim\limits_{x\to 3} \dfrac{x-3}{x^2-3}$

(h) $\displaystyle\lim_{x\to\infty} \frac{\ln x}{\sqrt{x}}$

(i) $\displaystyle\lim_{x\to 0} x \cot x$

(j) $\displaystyle\lim_{x\to 0} \left(\frac{\sin x}{x}\right)^{\frac{1}{x^2}}$

(k) $\displaystyle\lim_{x\to 0^+} (1+ax)^{\frac{b}{x}}$

(l) $\displaystyle\lim_{x\to 0^+} x^x$

2. Find a value of c that makes the function

$$f(x) = \begin{cases} \frac{9x-3\sin 3x}{5x^3} & \text{if } x \neq 0 \\ c & \text{if } x = 0 \end{cases}$$

continuous at $x = 0$

3. (a) The **sine integral** $Si(x) = \displaystyle\int_0^x \frac{\sin u}{u}\, du$ is a useful function in applied mathematics.

   Find $\displaystyle\lim_{x\to 0} \frac{Si(x)}{x}$

   (b) The **Fresnel cosine integral** $C(x) = \displaystyle\int_0^x \cos(u^2)\, du$ is used in the analysis of the diffraction of light.

   Find: $\displaystyle\lim_{x\to 0} \frac{C(x) - x}{x^5}$

4. If air resistance is disregarded, the acceleration of a falling object is given by $a(t) = g$, where g is a gravitational constant, so the velocity of an object t seconds after being released is given by: $v(t) = gt$.

   If we assume air resistance is proportional to the velocity v, it can be shown that the velocity of an object t seconds after being released is given by: $v(t) = \dfrac{mg}{k}\left(1 - e^{-kt/m}\right)$, where m is the mass of the object and k is a constant air-resistance coefficient. In the following problems, we will explore the consequences of taking air resistance into account when studying the velocity, $v(t)$ of a falling object.

   (a) Find $\lim\limits_{k\to 0^+} v(t)$

   [Consequently, $k = 0$ corresponds to the case of no air resistance.]

   (b) Find $\lim\limits_{t\to\infty} v(t)$

   [Consequently, a body's velocity tends to a finite limit, its *terminal velocity*, after it has fallen a sufficiently long time.]

   (c) Find $\lim\limits_{m\to 0^+} v(t)$

   [Consequently, a light "feathery" body falls very slowly through the air.]

   (d) Find $\lim\limits_{m\to\infty} v(t)$

   [Consequently, a heavy body tends to fall much as it would with no air resistance.]