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Let us prove that $G_{2860} \cong H = \langle s, t \mid s^2 = t^2 \rangle$. Indeed, the relation $a^2 = b^2$ is satisfied, so G_{2860} is a homomorphic image of H with respect to the homomorphism induced by $s \mapsto a$ and $t \mapsto b$. Each element of H can be written in the form $t^r (st)^l s^n$, $n \in \mathbb{Z}, l \geq 0, r \in \{0, 1\}$. It suffices to prove that images of these words (except for the identity word, of course) represent nonidentity elements in G_{2860}.

We have $a^{2n} = (a^n, a^n)$, $a^{2n+1} = \sigma(a^{n+1}, a^n)$, $(ab)^l = (1, a^{2l})$. We only need to check words of even length (those of odd length act nontrivially on level 1). We have $(ab)^\ell a^{2n} = (a^n, a^{n+2\ell}) \neq 1$ in G if $n \neq 0$ or $\ell \neq 0$, since a has infinite order. On the other hand, $b(ab)^l a^{2n+1} = (a^{n+1+2l+1}, a^n) = 1$ if and only if $n = 0$ and $l = -1$, which is not the case, because l must be nonnegative. This finishes the proof.

2861 $\cong G_{731} \cong \mathbb{Z}$. Wreath recursion: $a = \sigma(b, c)$, $b = \sigma(c, a)$, $c = (c, c)\rangle$.

Since c is trivial, $ba = (ab, 1)$, $ab = (1, ba)$, which yields $a = b^{-1}$. Also $a^{2n} = (b^n, b^n), b^{2n} = (a^n, a^n)$ and $a^{2n+1} \neq 1, b^{2n+1} \neq 1$. Thus a has infinite order and $G_{2861} \cong \mathbb{Z}$.

2862 $\cong G_{847} \cong D_4$. Wreath recursion: $a = \sigma(c, c)$, $b = \sigma(c, a)$, $c = (c, c)\rangle$.

Direct calculation.

2874 $\cong G_{820} \cong D_\infty$. Wreath recursion: $a = \sigma(a, c)$, $b = \sigma(b, b)$, $c = (c, c)$.

Since c is trivial, $G_{2874} = \langle b, ba \rangle$. Since $ba = (ba, b)$, the elements b and ba form a 2-state automaton generating D_∞ (see Theorem 7).

2880 $\cong G_{730}$. Klein Group $C_2 \times C_2$. Wreath recursion: $a = \sigma(c, c)$, $b = \sigma(b, b)$, $c = (c, c)$.

Direct calculation.

2887 $\cong G_{731} \cong \mathbb{Z}$. Wreath recursion: $a = \sigma(a, c)$, $b = \sigma(c, b)$, $c = (c, c)$.

Note that c is trivial, b is the adding machine and $a = b^{-1}$.

2889 $\cong G_{848} \cong C_2 \wr \mathbb{Z}$. Wreath recursion: $a = \sigma(c, c)$, $b = \sigma(c, b)$, $c = (c, c)$.

Note that c is trivial. Since b is the adding machine and $ab = (1, b)$, we have $G_{2889} = \langle b, ab \rangle = G_{848}$.

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Let us prove that $G_{2860} \cong H = \langle s, t \mid s^2 = t^2 \rangle$. Indeed, the relation $a^2 = b^2$ is satisfied, so G_{2860} is a homomorphic image of H with respect to the homomorphism induced by $s \mapsto a$ and $t \mapsto b$. Each element of H can be written in the form $t^r(st)^l s^n$, $n \in \mathbb{Z}, l \geq 0, r \in \{0, 1\}$. It suffices to prove that images of these words (except for the identity word, of course) represent nonidentity elements in G_{2860}.

We have $a^{2n} = (a^n, a^n)$, $a^{2n+1} = \sigma(a^{n+1}, a^n)$, $(ab)^l = (1, a^{2l})$. We only need to check words of even length (those of odd length act nontrivially on level 1). We have $(ab)^\ell a^{2n} = (a^n, a^{n+2\ell}) \neq 1$ in G if $n \neq 0$ or $\ell \neq 0$, since a has infinite order. On the other hand, $b(ab)^l a^{2n+1} = (a^{n+1+2l+1}, a^n) = 1$ if and only if $n = 0$ and $l = -1$, which is not the case, because l must be nonnegative. This finishes the proof.

2861 $\cong G_{731} \cong \mathbb{Z}$. Wreath recursion: $a = \sigma(b, c)$, $b = \sigma(c, a)$, $c = (c, c)\rangle$.

Since c is trivial, $ba = (ab, 1)$, $ab = (1, ba)$, which yields $a = b^{-1}$. Also $a^{2n} = (b^n, b^n), b^{2n} = (a^n, a^n)$ and $a^{2n+1} \neq 1, b^{2n+1} \neq 1$. Thus a has infinite order and $G_{2861} \cong \mathbb{Z}$.

2862 $\cong G_{847} \cong D_4$. Wreath recursion: $a = \sigma(c, c)$, $b = \sigma(c, a)$, $c = (c, c)\rangle$.

Direct calculation.

2874 $\cong G_{820} \cong D_\infty$. Wreath recursion: $a = \sigma(a, c)$, $b = \sigma(b, b)$, $c = (c, c)$.

Since c is trivial, $G_{2874} = \langle b, ba \rangle$. Since $ba = (ba, b)$, the elements b and ba form a 2-state automaton generating D_∞ (see Theorem 7).

2880 $\cong G_{730}$. Klein Group $C_2 \times C_2$. Wreath recursion: $a = \sigma(c, c)$, $b = \sigma(b, b)$, $c = (c, c)$.

Direct calculation.

2887 $\cong G_{731} \cong \mathbb{Z}$. Wreath recursion: $a = \sigma(a, c)$, $b = \sigma(c, b)$, $c = (c, c)$.

Note that c is trivial, b is the adding machine and $a = b^{-1}$.

2889 $\cong G_{848} \cong C_2 \wr \mathbb{Z}$. Wreath recursion: $a = \sigma(c, c)$, $b = \sigma(c, b)$, $c = (c, c)$.

Note that c is trivial. Since b is the adding machine and $ab = (1, b)$, we have $G_{2889} = \langle b, ab \rangle = G_{848}$.

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2889 $\cong G_{848} \cong C_2 \wr \mathbb{Z}$. Wreath recursion: $a = \sigma(c, c)$, $b = \sigma(c, b)$, $c = (c, c)$.

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2880 $\cong G_{730}$. Klein Group $C_2 \times C_2$. Wreath recursion: $a = \sigma(c, c)$, $b = \sigma(b, b)$, $c = (c, c)$.

Direct calculation.

2887 $\cong G_{731} \cong \mathbb{Z}$. Wreath recursion: $a = \sigma(a, c)$, $b = \sigma(c, b)$, $c = (c, c)$.

Note that c is trivial, b is the adding machine and $a = b^{-1}$.

2889 $\cong G_{848} \cong C_2 \wr \mathbb{Z}$. Wreath recursion: $a = \sigma(c, c)$, $b = \sigma(c, b)$, $c = (c, c)$.

Note that c is trivial. Since b is the adding machine and $ab = (1, b)$, we have $G_{2889} = \langle b, ab \rangle = G_{848}$.

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