2-generator one-relator group so that in order to prove the coherence of all one-relator groups, it is sufficient to consider only those with 2-generators. Next, these groups are particularly nice from the perspective of conformally CAT(0) structures on special polyhedra, since their Euler characteristics are zero. As a result, the only feasible conformal CAT(0) structures are those where the face curvatures are 0 and the vertex curvatures are 0. In other words, all the inequalities become equalities. Another less obvious consequence of the tightness of the constraints is that the pairs of non-adjacent angles in the link of a vertex must have the exact same angles assigned. Thus, if we let n denote the number of vertices in the complex, there are exactly $3n$ variables which determine the angle values. There are n linear equations associated with the face curvatures and n linear equations associated with the curvature of the vertex links. From a linear algebra perspective, so long as the system of $2n$ linear equations in $3n$ variables is consistent, there should be at least an n-dimensional family of solutions. Finally, the process of converting the standard 2-complex for a one-relator group into a special polyhedron is quite flexible. It is possible to insist that the result has no polygons with one or two sides and no "untwisted" triangles. Even at this point there are a number of moves which can be done which only slightly change the complex while significantly changing the system of linear equations. The net result is that the n vertex equations are always consistent with each other and the n face equations are almost always consistent each other.

Concretely, Noel Brady and I have also tried our hand at finding conformal CAT(0) structures for a half-a-dozen examples of 2-generator one-relator groups chosen at random. Starting from a randomly chosen relator of length 12 or so, we haphazardly converted it to a special polyhedron using the pineapple and banana tricks among others [50]. Once we had a special polyhedron, we then worked to remove all monogons and bigons. At this point we stopped and found the linear system of equations which needed to be satisfied and we tested whether this system had a solution. In all cases, it did, and in all but one case, the resulting angle assignment could be extended to a piecewise Euclidean metric on the 2-complex. In other words, the results were typically CAT(0) in addition to being conformally CAT(0). The positive evidence is, admittedly, rather meager at the moment. A computer program to check all of the 2-generator one-relator groups out to a modest size would be more convincing and is currently under development. A general program for investigating conformal CAT(0) structures on special polyhedra for 2-generator one-relator groups is also being actively pursued. In addition to being a reasonable approach to the coherence question for one-relator groups, even partial progress on this geometric conjecture would certainly help to explain why one-relator groups have such a tendency to act like non-positively curved groups, even though it is well-known that they are not non-positively curved in general.

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