## Math 127 - College Algebra

#### Handout: Properties of Exponents and Radicals

# A. Exponents

**Definition:**  $a^n = a \cdot a$  (a multiplied by itself n times)

# Properties:

1. 
$$a^0 = 1$$

2. 
$$a^{-n} = \frac{1}{a^n}$$

$$3. \ a^m \cdot a^n = a^{m+n}$$

4. 
$$(a^m)^n = a^{mn}$$

5. 
$$(ab)^n = a^n b^n$$

6. 
$$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$$

7. 
$$\frac{a^m}{a^n} = a^{m-n} = \frac{1}{a^{n-m}}$$

8. 
$$\frac{a^{-m}}{b^{-n}} = \frac{b^n}{a^m}$$

9. 
$$\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n$$

## B. Radicals:

**Definition:** Suppose n is a positive integer and a is a real number. Then we define the nth root of a, denoted by  $\sqrt[n]{a}$  as follows:

- If a = 0, then  $\sqrt[n]{a} = 0$ .
- If a > 0 then  $\sqrt[n]{a}$  is the positive real number b such that  $b^n = a$ .
- If a < 0 and n is **odd**, then  $\sqrt[n]{a}$  is the negative real number b such that  $b^n = a$ .
- If a < 0 and n is **even**, then  $\sqrt[n]{a}$  is not a real number, since there is no real number b such that  $b^n = a$ .

# Examples:

(a) 
$$\sqrt[2]{9} = \sqrt{9} = 3$$
 since  $3 \cdot 3 = 9$ .

(b) 
$$\sqrt[3]{-8} = -2$$
 since  $(-2) \cdot (-2) \cdot (-2) = -8$ .

(c)  $\sqrt{-16}$  is not a real number. (notice that  $4 \cdot 4 = 16$ , and  $(-4) \cdot (-4) = 16$ )

#### **Properties:**

1. 
$$(\sqrt[n]{a})^n = a$$
 if  $\sqrt[n]{a}$  is a real number.

$$2. \ \sqrt[n]{a^n} = a \text{ if } a \ge 0.$$

3. 
$$\sqrt[n]{a^n} = a$$
 if  $a < 0$  and  $n$  is odd.

4. 
$$\sqrt[n]{a^n} = |a|$$
 if  $a < 0$  and n is even.

5. 
$$\sqrt[n]{ab} = \sqrt[n]{a} \sqrt[n]{b}$$
 provided both exist.

6. 
$$\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$$
 provided both exist.

7. 
$$\sqrt[m]{\sqrt[n]{a}} = \sqrt[mn]{a}$$
 provided both exist.

#### Warning!!

(a) In general, 
$$\sqrt{a^2 + b^2} \neq a + b$$

(b) Also, in general, 
$$\sqrt{a+b} \neq \sqrt{a} + \sqrt{b}$$

### **Exponents and Radicals:**

1. 
$$\sqrt[n]{a} = a^{\frac{1}{n}}$$
.

2. 
$$\sqrt[n]{a^m} = a^{\frac{m}{n}} = \left(a^{\frac{1}{n}}\right)^m = (a^m)^{\frac{1}{n}}$$
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