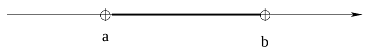
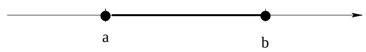
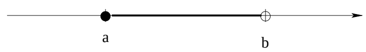
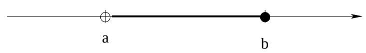
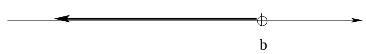
Math 127
Inequalities

A. Interval Notation

Finite Intervals:

Interval	Inequality	Graph
Open: (a, b)	$a < x < b$	
Closed: $[a, b]$	$a \leq x \leq b$	
Half Open: $[a, b)$	$a \leq x < b$	
$(a, b]$	$a < x \leq b$	

Infinite Intervals:

Interval	Inequality	Graph
(a, ∞)	$a < x$	
$[a, \infty)$	$a \leq x$	
$(-\infty, b)$	$x < b$	
$(-\infty, b]$	$x \leq b$	

B. Properties of Inequalities

1. If $a < b$ and $b < c$ then $a < c$ [Transitivity]
2. If $a < b$, then $a + c < b + c$ [Additive Shift]
3. If $a < b$ and $c > 0$, then $ac < bc$ [Positive Multiplication]
4. If $a < b$ and $c < 0$, then $ac > bc$ [Negative Multiplication - Inequality Reversal!]

C. Solving Inequalities

Our goal is to use the properties of inequalities and other algebraic techniques to find which real numbers satisfy a given inequality.

Examples:

1. $3 - 2x < 5$

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  3 - 2x <  5
 -3         -3
    - 2x < 2
      \-2  \-2
       x > -1
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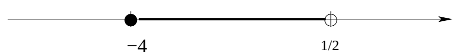
2. $3 < 4 - 2x \leq 12$

$$3 < 4 - 2x \leq 12$$
$$-4 \;\; -4 \qquad\quad -4$$
$$-1 < -2x \quad \leq 8$$
$$\div -2 \; \div -2 \quad \div -2$$
$$\tfrac{1}{2} > \; x \quad \geq -4$$



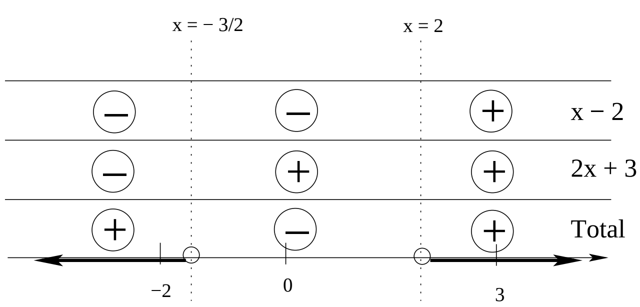
3. $2x^2 - x - 6 > 0$ – for this one, we will need to use "sign analysis"

$(2x+3)(x-2) > 0$

Solving the related linear equations:

$2x + 3 = 0 \;\rightarrow\; x = -\frac{3}{2}$

$x - 2 = 0 \;\rightarrow\; x = 2$



Therefore, the solution to this inequality is: $(-\infty, -\frac{3}{2}) \cup (2, \infty)$