

## Quadratic Equations

**Definition:** A **quadratic equation** is an equation that can be written in the form  $ax^2 + bx + c = 0$  with  $a \neq 0$ .

### Methods for Solving Quadratic Equations:

#### 1. Factoring [this only works if the quadratic happens to factor]

Example:  $3x^2 + 5x - 2 = 0$

- (Factor)  $(3x - 1)(x + 2) = 0$  [this only works if one side of the equation is 0!]
- (Split)  $3x - 1 = 0$  or  $x + 2 = 0$
- (Solve)  $3x = 1$ , so  $x = \frac{1}{3}$  or  $x = -2$ .

#### 2. Special Form [this works for quadratics of the form $a(x - h)^2 = d$ ]

Example:  $4(x - 5)^2 = 13$

- (divide) [if necessary]  $(x - 5)^2 = \frac{13}{4}$
- (square root)  $x - 5 = \pm\sqrt{\frac{13}{4}}$
- (add and simplify) [if necessary]  $x = 5 \pm \frac{\sqrt{13}}{2}$

#### 3. Completing the Square [this always works]

Example:  $2x^2 - 4x - 5 = 0$

- (move constant)  $2x^2 - 4x = 5$
- (divide by  $a$  if  $a \neq 1$ )  $x^2 - 2x = \frac{5}{2}$
- (add the constant  $(\frac{b}{2})^2$  to form a perfect square)  $(\frac{b}{2}) = (-\frac{2}{2})^2 = 1$   
so we have:  $x^2 - 2x + 1 = \frac{5}{2} + 1$
- (factor)  $(x - 1)^2 = \frac{7}{2}$
- (square root)  $x - 1 = \pm\sqrt{\frac{7}{2}}$
- (add and simplify)  $x = 1 \pm \frac{\sqrt{7}}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = 1 \pm \frac{\sqrt{14}}{2}$

#### 4. The Quadratic Formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

**Proof:** (we will use completing the square to derive this formula)

$$ax^2 + bx + c = 0$$

$$ax^2 + bx = -c$$

$$x^2 + \frac{b}{a}x = -\frac{c}{a}$$

$$x^2 + \frac{b}{a}x + \left(\frac{b}{2a}\right)^2 = -\frac{c}{a} + \left(\frac{b}{2a}\right)^2$$

$$\left(x + \frac{b}{2a}\right)^2 = -\frac{c}{a} + \frac{b^2}{4a^2}$$

$$x + \frac{b}{2a} = \pm\sqrt{-\frac{c}{a} + \frac{b^2}{4a^2}} = \pm\sqrt{\frac{-4ac}{4a^2} + \frac{b^2}{4a^2}} = \pm\sqrt{\frac{-4ac + b^2}{4a^2}}$$

$$x = -\frac{b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Example:  $5x^2 - 3x - 1$  [so  $a = 5$ ,  $b = -3$ , and  $c = -1$ ]

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{3 \pm \sqrt{9 - 4(5)(-1)}}{2(5)} = \frac{3 \pm \sqrt{9 + 20}}{10}$$

$$= \frac{3 \pm \sqrt{29}}{10}$$