Math 487
Chapter 1 Quiz - Practice Problems

1. Explain each of the following in your own words:
   (a) What is an axiomatic system?
   (b) What is required for an axiomatic system to be consistent?
   (c) What is required for an axiomatic system to be independent?
   (d) What is required for an axiomatic system to be complete?
   (e) What is required for an axiomatic system to satisfy the principle of duality?
2. Is there a projective plane of order 1? If there is, how many points and lines are in the geometry? If not, why not?
3. Is there a projective plane of order 11? If there is, how many points and lines are in the geometry? If not, why not?
4. Consider the following axiomatic system:
   $A1$: For any two distinct points, there is exactly one line incident with both points.
   $A2$: For any two distinct lines, there is at most one point incident with both lines.
   $A3$: Every line is incident with at least two points.
   $A4$: There are 5 points in this geometry.
   $A5$: No point is incident with every line.
   (a) State the undefined terms in this axiomatic system.
   (b) Find a model for this geometry. Be sure to explain how you know that each axiom holds.
   (c) Is this axiomatic system consistent? Justify your answer.
   (d) Find a second model for this geometry that is not isomorphic to your previous model. Be sure to explain how you know that each axiom holds and how you know this it is not isomorphic to your previous model.
   (e) Is this axiomatic system complete? Justify your answer.
   (f) Which of the axioms in this model are independent? Justify your answer.
   (g) Prove the theorem $T1$: There are at least five lines in this geometry.
   (h) Write the duals to $A3$ and $A5$. Are these dual statements true statements in this axiomatic system?
   (i) Does this axiomatic system satisfy the principle of duality? Why or why not?