

Math 487
Exam 2 Review Sheet

Postulates: Memorize the names and statements of each of the following SMSG postulates.

- Postulate 1: (Line Uniqueness) Given any two distinct points there is exactly one line that contains them.
- Postulate 2: (Distance Postulate) To every pair of distinct points there corresponds a unique positive number. This number is called the distance between the two points.
- Postulate 3: (Ruler Postulate) The points of a line can be placed in a correspondence with the real numbers such that:
 - To every point of the line there corresponds exactly one real number.
 - To every real number there corresponds exactly one point of the line.
 - The distance between two distinct points is the absolute value of the difference of the corresponding real numbers.
- Postulate 4: (Ruler Placement Postulate) Given two points P and Q of a line, the coordinate system can be chosen in such a way that the coordinate of P is zero and the coordinate of Q is positive.
- Postulate 5a: (Existence of Points) Every plane contains at least three non-collinear points.
- Postulate 9: (Plane Separation Postulate) Given a line and a plane containing it, the points of the plane that do not lie on the line form two sets such that:
 - each of the sets is convex
 - if P is in one set and Q is in the other, then segment \overline{PQ} intersects the line.
- Postulate 11: (Angle Measurement Postulate) To every angle there corresponds a real number between 0 and 180.
- Postulate 12: (Angle Construction Postulate) Let \overrightarrow{AB} be a ray on the edge of the half-plane H . For every r between 0 and 180, there is exactly one with P in H such that $m(\angle PAB) = r$.
- Postulate 13: (Angle Addition Postulate) If D is a point in the interior of $\angle BAC$, then $m(\angle BAC) = m(\angle BAD) + m(\angle DAC)$.
- Postulate 14: (Supplement Postulate) If two angles form a linear pair, then they are supplementary.
- Postulate 15: (SAS Postulate) Given a one-to-one correspondence between two triangles (or between a triangle and itself). If two sides and the included angle of the first triangle are congruent to the corresponding parts of the second triangle, then the correspondence is a congruence.
- Postulate 16: (Parallel Postulate) Through a given external point there is at most one line parallel to a given line.

Notes:

- You should also know the statements of the Hyperbolic and Elliptic Parallel Postulates.
- You should also know which axioms are not independent of the other axioms.
- You should know the impact of the various parallel postulates on triangle angle sums, triangle congruence, rectangles, summit angles of Saccheri quadrilaterals, and triangle similarity.

Models: You should be familiar with each of the following components of geometric models.

- Understand how to compute distances in Discrete Planes, The Riemann and Modified Riemann Spheres, The Euclidean Plane, The Taxicab Plane, the Max-Distance Plane, the Missing Strip Plane, and the Poincare Half-Plane.
- Be able to determine whether or not a given function is a distance.
- Understand how lines are defined in each of the models above.
- Given two points, be able to find the distance between the points and the equation of the line containing the two points in each of the models.
- Know the standard ruler in the Euclidean Plane and the Taxicab Plane.
- Be prepared to discuss which models satisfy a given SMSG postulate.

Definitions: You should be familiar with the definitions of the following terms.

- ruler, one to one, onto, line segment, ray, betweenness, midpoint
- equivalence relation, congruence of segments and angles
- the interior of an angle, right, acute, and obtuse angles, parallel and perpendicular lines
- angle bisector, perpendicular bisector, linear pair, supplementary angles, triangle congruence
- exterior angles, remote interior angles, transversal, alternate interior angles, Saccheri Quadrilateral, parallelogram, rectangle

Theorems: You should be familiar with the **statements** of the following Theorems.

- Theorem 2.6(Pasch's Postulate), Theorem 2.7, Theorem 2.9(The Crossbar Theorem), Theorem 2.11(Exterior Angle Theorem), The SSS Theorem, Theorem 2.20

Theorems: You should be familiar with the **statements and proofs** of the following Theorems.

- Theorem 2.2, Theorem 2.8(The Vertical Angle Theorem), Theorem 2.10(Pons Asinorum), The ASA Theorem, Theorem 2.16, Theorem 2.17

Other Proofs:

For the exam I will expect you to be able to:

- Prove 1 or 2 of the Euclidean Propositions from your text. You do not need to memorize them. I will provide the statements and will expect you to prove each theorem using postulates and other theorems from the course.
- Fill in the reasons for each step in the two column proof using postulates and theorems listed above.