Math 487
Final Exam Review Sheet

Part I: Projective Geometry

• Memorize the six axioms for a Plane Projective Geometry
• Know the definitions of the terms:
collinear points, concurrent lines, a complete quadrangle (vertices, sides, opposite sides, and diagonal points), a complete quadrilateral, a triangle, a pencil of points, a pencil of lines, perspectivity from a point, perspectivity from a line, a center of perspectivity, an axis of perspectivity, a projectivity, an elementary correspondence, a harmonic set, the harmonic conjugate of a point, a point conic, and a line conic.
• Know that the six axioms of a Plane Projective Geometry are independent, and be able to show that the first four axioms are independent.
• Know that a Plane Projective Geometry satisfies the principle of duality. Be able to state the dual of any axiom and any theorem in this geometry. Also be able to prove the duals of axioms 3, 4, and 5.
• Know the statement of each of the following theorems: Theorem 4.4, Theorem 4.7, Theorem 4.8, Theorem 4.10, Theorem 4.11 (FT of PG), Corollary 4.12, Theorem 4.13.
• Know the statement and the proof of each of the following theorems: Theorem 4.1, Theorem 4.2, Theorem 4.3, Theorem 4.5, Theorem 4.6.
• Be able to construct a harmonic set containing three given collinear points. Also be able to construct a harmonic set representing a given chord by computing the length ratios from the frequency ratios of the notes in the chord.
• Be able to find the image of a point or a line under a projectivity. Also be able to construct a projectivity between two given pencils of lines and/or points.

Part IIa: Axiomatic Systems

• Know the definitions of: an axiomatic system, a model for an axiomatic system, a consistent system of axioms, a complete system of axioms.
• Be able to determine whether or not a model satisfies the axioms of a given axiomatic system.
• Understand what it means for an axiomatic system to be independent and be able to determine whether or not a given axiomatic system is independent.
• Given two models, be able to determine whether or not the models are isomorphic.
• Understand the principle of duality and be able to find the dual of a given axiom.
• Given an axiomatic system, be able to prove theorems about the axiomatic system and be able to find one or more models for the given system.
• Given the Axioms for Fano's Geometry, be able to prove theorems about this geometry.
• Know that there are finite projective geometries of order p^m for any prime number p and positive integer m.
• Know that a projective plane of order n has exactly $n^2 + n + 1$ points and exactly $n^2 + n + 1$ lines.

Part IIb: Euclidean and Non-Euclidean Geometry

• Memorize the names and statements of SMSG postulates 1, 2, 3, 4, 5a, 9, 11, 12, 13, 14, 15, and 16.
• Know the statement of Playfair's Axiom, and the statements of the Hyperbolic and Elliptic Parallel Postulates.
• Know the impact of the various parallel postulates on triangle angle sums, triangle congruence, rectangles, summit angles of Saccheri quadrilaterals, and triangle similarity.
• Understand the definition of the following models: The Riemann and Modified Riemann Spheres, The Euclidean Plane, The Taxicab Plane, the Max-Distance Plane, the Missing Strip Plane, and the Poincare Half-Plane.
• Understand how points, lines, and distance are defined in each of the models above.
• Be prepared to discuss which models satisfy a given SMSG postulate.
• Know the following definitions: ruler, one to one, onto, line segment, ray, betweenness, midpoint, equivalence relation, congruence of segments and angles, the interior of an angle, right, acute, and obtuse angles, parallel and perpendicular lines, angle bisector, perpendicular bisector, linear pair, supplementary angles, triangle congruence, exterior angles, remote interior angles, transversal, alternate interior angles, a Saccheri Quadrilateral, a parallelogram, and a rectangle.

• You should be familiar with the **statements** of the following Theorems.

- Theorem 2.6(Pasch's Postulate), Theorem 2.7, Theorem 2.9(The Crossbar Theorem), Theorem 2.11(Exterior Angle Theorem), The SSS Theorem, Theorem 2.20

• You should be familiar with the **statements and proofs** of the following Theorems.

- Theorem 2.2, Theorem 2.8(The Vertical Angle Theorem), Theorem 2.10(Pons Asinorum), The ASA Theorem, Theorem 2.16, Theorem 2.17

• Be able to prove a Euclidean Proposition from your text. You do not need to memorize them. I will provide the statement and will expect you to prove the theorem using postulates and other theorems from the course.

Part IIc: Transformational Geometry

• Know the definitions of: a mapping, domain, co-domain, range, one-to-one, onto, a transformation, a transformation of a plane, and a group.
• Be able to determine whether or not a given binary operation on a set is a group.
• Know (and be able to prove) which sets of transformations form a group under function composition.

• Understand the Analytic Model of $\mathbb{E}$, including:

- how to find the homogeneous coordinates of points and lines in this model
- how to use matrix operations to determine whether or not a point is on a given line
- Be able to find the line containing a given pair of points and be able to find the point of intersection of a pair of non-parallel lines.
- Know how to compute distances and angles in this model.

• Know the definition and standard matrix representation of an affine transformation.
• Know the definition of and the standard matrix representations of direct and indirect isometries.
• Know that the determinant of a direct isometry is 1 and the determinant of an indirect isometry is -1.
• Understand the impact of composition (matrix multiplication) on pairs of direct and/or indirect isometries.
• Know and be able to either prove or apply the fact that betweenness, collinearity, lines, segments, rays, circles, congruent triangles, and angle measure are all preserved under isometry.
• Know and be able to apply the fact that an isometry is completely determined by its action on three non-collinear points.
• Be able to determine whether or not a given function is: a transformation, an isometry.
• Know the definition of and the general matrix form for translations, rotations, and reflection in $\mathbb{E}$ and be able to find the matrix representation of given specific isometry. Also be able to classify a transformation given in terms of a 3×3 matrix.
• Given a plane figure, be able to draw its image under a given isometry.
• Know the definition of the symmetries of a set of points and be able to investigate which isometries are in the group of symmetries of a given set of points.