Math 127
Exam 1 Practice Problems

1. True or False:

   (a) $(a+b)c = ac+bc$
   Solution: True (This is one of the forms of the distributive property).

   (b) If $ab = 1$, then either $a = 1$ or $b = 1$ or both a and b equal 1
   Solution: False. For example, if $a = \frac{1}{2}$ and $b = 2$, then $ab = 1$.

   (c) $\frac{a}{b} + \frac{c}{d} = \frac{a+c}{b+d}$
   Solution: False. For example, if $a = c = 3$, and $b = d = 1$, then $\frac{a}{b} + \frac{c}{d} = \frac{3}{1} + \frac{3}{1} = 6$, while $\frac{a+c}{b+d} = \frac{3+3}{1+1} = \frac{6}{2} = 3$.

   (d) $\frac{a+c}{b} = \frac{a}{b} + \frac{c}{b}$
   Solution: True

   (e) $5^{\frac{1}{2}} = \frac{1}{5^2}$
   Solution: False. $5^{\frac{1}{2}} = \sqrt{5}$, while $\frac{1}{5^2} = \frac{1}{25}$.

   (f) $(a+b)^2 = a^2 + b^2$
   Solution: False. $(a+b)^2 = (a+b)(a+b) = a^2 + 2ab + b^2$.

2. Rationalize the denominator in the following expressions:

   (a) $\frac{3x}{\sqrt[3]{x}}$
   Solution:
   $\frac{3x}{\sqrt[3]{x}} \cdot \frac{\sqrt[3]{x^2}}{\sqrt[3]{x^2}} = \frac{3x\sqrt[3]{x^2}}{x} = 3\sqrt[3]{x^2}$

   (b) $\frac{2x+3}{\sqrt{2x}-1}$
   Solution:
   $\frac{2x+3}{\sqrt{2x}-1} \cdot \frac{\sqrt{2x}+1}{\sqrt{2x}+1} = \frac{(2x+3)\sqrt{2x}+1}{(2x-1)}$

3. Perform the indicated operations and simplify:

   (a) $3(2x^3 - x^2 + 5x) - 2x(3x^3 - 2x^2 + 5x - 3)$
   Solution:
   $3(2x^3-x^2+5x)-2x(3x^3-2x^2+5x-3) = 6x^3-3x^2+15x-6x^4+4x^3-10x^2+6x = -6x^4+10x^3-13x^2+21x$

   (b) $(2x^2 + 3x - 2)(x - 2)$
   Solution:
   $(2x^2 + 3x - 2)(x - 2) = 2x^3 + 3x^2 - 2x - 4x^2 - 6x + 4 = 2x^3 - x^2 - 8x + 4$

   (c) $(2x+1)^3$
   Solution:
   $(2x+1)^3 = (2x+1)(2x+1)(2x+1) = (4x^2+4x+1)(2x+1) = 8x^3+4x^2+8x^2+4x+2x+1 = 8x^3+12x^2+6x+1$, or, use the expansion formula for cubes, which yields the same result.

   (d) $(x^{\frac{1}{3}} + y^{\frac{1}{3}})(x^{\frac{2}{3}} - x^{\frac{1}{3}}y^{\frac{1}{3}} + y^{\frac{2}{3}})$
   Solution:
   $= x + x^{\frac{2}{3}}y^{\frac{1}{3}} - x^{\frac{2}{3}}y^{\frac{1}{3}} - x^{\frac{1}{3}}y^{\frac{2}{3}} + x^{\frac{1}{3}}y^{\frac{2}{3}} + y$
   $= x + y$

4. Factor each of the following expressions completely:

(a) $2x^2 + x - 6$
Solution: $(2x-3)(x+2)$

(b) $50x^2 + 45x - 18$
Solution:
$(50 \cdot -18 = -900 = -1 \cdot 10 \cdot 10 \cdot 9 = -1 \cdot 5 \cdot 5 \cdot 2 \cdot 2 \cdot 3 \cdot 3 = 60 \cdot -15)$
$= 50x^2 + 60x - 15x - 18 = 10x(5x+6) - 3(5x+6) = (5x+6)(10x-3)$

(c) $9x^2 - 49y^6$
Solution: $(3x - 7y^3)(3x + 7y^3)$

(d) $8x^3 - y^3$
Solution: $(2x - y)(4x^2 + 2xy + y^2)$

(e) $6x^3y - 27x^2y - 15xy$
Solution: $(3xy)(2x^2 - 9x - 5) = (3xy)(2x+1)(x-5)$

(f) $3x^3 + x^2 - 3x - 1$
Solution: $x^2(3x+1) - 1(3x+1) = (x^2-1)(3x+1) = (x+1)(x-1)(3x+1)$

(g) $x^6 - 1$
Solution:
$x^6 - 1 = (x^2)^3 - (1)^3$, so by the difference of cubes factoring formula:
$= (x^2-1)(x^4+x^2+1)$
$= (x+1)(x-1)(x^4+x^2+1)$

5. Simplify the following expressions:

(a) $\dfrac{3x^2-10x+3}{x^2-1} \cdot \dfrac{x^2+x-2}{x^2-9}$
Solution:
$$\frac{3x^2-10x+3}{x^2-1} \cdot \frac{x^2+x-2}{x^2-9} = \frac{(3x-1)(x-3)}{(x+1)(x-1)} \cdot \frac{(x+2)(x-1)}{(x+3)(x-3)} = \frac{(3x-1)(x+2)}{(x+1)(x+3)}$$

(b) $\dfrac{2x^2+4}{2x^2+7x-4} - \dfrac{x-1}{x+4}$
Solution:
$$\frac{2x^2+4}{2x^2+7x-4} - \frac{x-1}{x+4} = \frac{2x^2+4}{(2x-1)(x+4)} - \frac{x-1}{x+4} = \frac{2x^2+4}{(2x-1)(x+4)} - \frac{(2x-1)(x-1)}{(2x-1)(x+4)}$$
$$= \frac{2x^2+4-(2x^2-3x+1)}{(2x-1)(x+4)} = \frac{2x^2+4-2x^2+3x-1}{(2x-1)(x+4)} = \frac{3x+3}{(2x-1)(x+4)} = \frac{3(x+1)}{(2x-1)(x+4)}$$

(c) $\dfrac{\frac{1}{x}+\frac{3}{x-2}}{\frac{4}{x-1}-\frac{2}{x-2}}$
Solution:
$$\frac{\frac{1}{x}+\frac{3}{x-2}}{\frac{4}{x-1}-\frac{2}{x-2}} \cdot \frac{x(x-1)(x-2)}{x(x-1)(x-2)}$$
$$= \frac{(x-1)(x-2)+3(x)(x-1)}{4(x)(x-2)-2(x)(x-1)} = \frac{(x-1)(x-2+3x)}{x[4(x-2)-2(x-1)]}$$
$$= \frac{(x-1)(4x-2)}{x[4x-8-2x+2]} = \frac{2(x-1)(2x-1)}{x(2x-6)} = \frac{2(x-1)(2x-1)}{2x(x-3)} = \frac{(x-1)(2x-1)}{x(x-3)}$$

(d) $\dfrac{\frac{3}{2x+2h+1}-\frac{3}{2x+1}}{h}$
Solution:
$$\frac{\frac{3}{2x+2h+1}-\frac{3}{2x+1}}{h} = \frac{\frac{3(2x+1)}{(2x+2h+1)(2x+1)}-\frac{3(2x+2h+1)}{(2x+1)(2x+2h+1)}}{h}$$

$$= \frac{(6x+3)-(6x+6h+3)}{(2x+2h+1)(2x+1)} \cdot \frac{1}{h} = \frac{-6h}{(2x+1)(2x+2h+1)(h)}$$
$$= \frac{-6}{(2x+1)(2x+2h+1)}$$

6. True or False:

   (a) Every quadratic equation has two distinct real solutions.
   Solution: False. For example, $x^2+2x+1=0$ has only one solution $(x^2+2x+1=(x+1)^2)$

   (b) $\sqrt{i^4}=-1$
   Solution: False. Since $i^4=1$, $\sqrt{i^4}=1$, not -1. If you tried to simplify by removing i^2 from under the radical and got -1, you forgot that you must take the absolute value of expressions removed from under an even root.

   (c) $x=0$ is a solution to the equation $x(x-2)=4$
   Solution: False. Notice that $0(0-2)=(0)(-2)=0\neq 4$.

7. Use completing the square to solve the quadratic equation $3x^2-12x+5=0$.

   Solution:

   $3x^2-12x=-5$

   $x^2-4x=-\frac{5}{3}$

   $x^2-4x+\left(-\frac{4}{2}\right)^2=-\frac{5}{3}+\left(-\frac{4}{2}\right)^2$

   $x^2-4x+4=-\frac{5}{3}+4=-\frac{5}{3}+\frac{12}{3}=\frac{7}{3}$

   $(x-2)^2=\frac{7}{3}$

   $x-2=\pm\frac{\sqrt{7}}{\sqrt{3}}$

   $x=2\pm\frac{\sqrt{21}}{3}$

8. Solve the following quadratic equations:

   (a) $4x^2-5x+10=2x^2-8x+12$
   Solution:
   Moving all the terms to the same side, we get $2x^2+3x-2=0$.
   Factoring, we then have $(2x-1)(x+2)=0$
   Therefore, we either have $(2x-1)=0$, in which case, $2x=1$, so $x=\frac{1}{2}$, or $x+2=0$, so $x=-2$.

   (b) $3x^2+10=5x$
   Solution:
   Moving all the terms to one side, we get $3x^2-5x+10=0$. This does not factor, so we proceed by using the quadratic formula with $a=3, b=-5$, and $c=10$.
   Therefore, $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}=\frac{5\pm\sqrt{25-(4)(3)(10)}}{(2)(3)}=\frac{5\pm\sqrt{25-120}}{6}=\frac{5\pm\sqrt{-95}}{6}=\frac{5}{6}\pm\frac{\sqrt{(-1)(19)(5)}}{6}=\frac{5}{6}\pm\frac{i\sqrt{95}}{6}$.

9. Perform the indicated operations and express your answer in the form $a+bi$:

   (a) $(3-2i)-(12+6i)$
   Solution: $(3-2i)-(12+6i)=(3-12)+(-2i-6i)=-9-8i$

   (b) $(3-2i)(5+3i)$
   Solution: $(3-2i)(5+3i)=15-10i+9i-6i^2=15+6-i=21-i$.

   (c) $\dfrac{3-2i}{3i}$
   Solution: $\dfrac{3-2i}{3i}=\dfrac{(3-2i)\cdot i}{(3i)\cdot i}=\dfrac{3i-2i^2}{3i^2}=\dfrac{3^i+2}{-3}=-\dfrac{2}{3}-i$

(d) $\dfrac{3-2i}{2+3i}$

Solution: $\dfrac{3-2i}{2+3i} = \dfrac{(3-2i)(2-3i)}{(2+3i)(2-3i)} = \dfrac{6-4i-9i+6i^2}{4+6i-6i-9i^2} = \dfrac{6-13i+6i^2}{4-9i^2} = \dfrac{6-13i-6}{4+9}$
$= \dfrac{-13i}{13} = -i$

(e) i^{3147}

Solution: Notice $3147 = 4(786)+3$, so $i^{3147} = (i^4)^{786} i^3 = 1^{786} i^3 = i^3 = -i$.

10. Solve the following equations:

(a) $7x + 2 = -12$

Solution: $7x = -14$, so $x = -2$

(b) $4(x-1) + 3(2-x) = 10$

Solution: $4x - 4 + 6 - 3x = 10$, or $x + 2 = 10$

Therefore, $x = 8$

(c) $\dfrac{3}{10}x - \dfrac{3}{5} = \dfrac{3}{2}$

Solution: Multiplying to clear the denominators, $10 \cdot \left[\dfrac{3}{10}x - \dfrac{3}{5}\right] = 10 \cdot \left[\dfrac{3}{2}\right]$

$3x - 6 = 15$, or $3x = 21$. Hence $x = 7$

(d) $-20x = 5x^2$

Solution: Collecting all terms on one side, $5x^2 + 20x = 0$.

Factoring, $5x(x+4) = 0$, so either $5x = 0$, or $x + 4 = 0$

Thus $x = 0$, or $x = -4$

(e) $(x+6)(x-2) = -7$

Solution: Notice that we must one again collect all terms on one side.

Multiplying, $x^2 + 6x - 2x - 12 = -7$, so, combining terms, $x^2 + 4x - 5 = 0$

Factoring, $(x+5)(x-1) = 0$, so either $x + 5 = 0$ or $x - 1 = 0$

Thus $x = -5$, or $x = 1$

(f) $\dfrac{3}{x+6} - \dfrac{1}{x-2} = \dfrac{-6}{x^2+4x-12}$

Solution: Factoring, $\dfrac{3}{x+6} - \dfrac{1}{x-2} = \dfrac{-6}{(x+6)(x-2)}$

Multiplying to clear the denominators, $(x+6)(x-2) \cdot \left[\dfrac{3}{x+6} - \dfrac{1}{x-2}\right] = (x+6)(x-2) \cdot \left[\dfrac{-6}{(x+6)(x-2)}\right]$

$= 3(x-2) - (x+6) = -6$, or $3x - 6 - x - 6 + 6 = 0$.

Then $2x - 6 = 0$, or $2x = 6$. Therefore, $x = 3$.

(g) $|2 - 3x| - 3 = 5$

Note that it is a good idea to isolate the absolute value expression before splitting into two cases.

$|2 - 3x| = 8$, so we have two possible cases:

$2 - 3x = 8$ or $2 - 3x = -8$

Then $-3x = 6$ or $-3x = -10$

So either $x = -2$ or $x = \frac{10}{3}$

(h) $x^4 - x^3 - 9x^2 + 9x = 0$

Solution: Notice that $x^4 - x^3 - 9x^2 + 9x = x(x^3 - x^2 - 9x + 9) = x[x^2(x-1) - 9(x-1)]$
$= x(x^2 - 9)(x-1) = x(x+3)(x-3)(x-1) = 0$.

Therefore we must have $x = 0$, $x + 3 = 0$, $x - 3 = 0$, or $x - 1 = 0$

Hence $x = 0, x = -3, x = 3$, and $x = 1$ are the solutions to this equation.

(i) $8x - x^{\frac{5}{3}} = 0$

Solution: $8x - x^{\frac{5}{3}} = x(8 - x^{\frac{2}{3}}) = 0$, so either $x = 0$ or $8 - x^{\frac{2}{3}} = 0$.

If $8 - x^{\frac{2}{3}} = 0$, then $8 = x^{\frac{2}{3}}$, so $(8)^{\frac{3}{2}} = \left(x^{\frac{2}{3}}\right)^{\frac{3}{2}}$. So $x = (8)^{\frac{3}{2}} = \sqrt{8^3} = 8\sqrt{8} = 16\sqrt{2}$.

Hence, our solutions are $x = 0$ or $x = 16\sqrt{2}$. (Notice that both of these solutions check: $8(0) - 0^{\frac{5}{3}} = 0$, and $8(16\sqrt{2}) - (16\sqrt{2})^{\frac{5}{3}} = 8 \cdot (8)^{\frac{3}{2}} - ((8)^{\frac{3}{2}})^{\frac{5}{3}} = 8^{\frac{5}{2}} - 8^{\frac{5}{2}} = 0$).

(j) $2 - \sqrt[3]{2x + x^2} = 0$

Solution:

Isolating the radical, we obtain $2 = \sqrt[3]{2x + x^2}$.

Cubing both sides, we then have $2^3 = \left(\sqrt[3]{2x + x^2}\right)^3$, or $8 = 2x + x^2$.

Moving everything to one side gives $x^2 + 2x - 8 = 0$, which factors as $(x + 4)(x - 2) = 0$.

Therefore, $x = -4$ and $x = 2$ are our potential solutions.

Checking these: $\sqrt[3]{2(-4) + (-4)^2} = \sqrt[3]{-8 + 16} = \sqrt[3]{8} = 2$, and $2 - 2 = 0$,

while $\sqrt[3]{2(2) + 2^2} = \sqrt[3]{4 + 4} = \sqrt[3]{8} = 2$, and $2 - 2 = 0$.

(k) $x + 5 = \sqrt{2x + 13}$

Solution: Squaring both sides: $(x + 5)^2 = 2x + 13$, or $x^2 + 10x + 25 = 2x + 13$

Therefore, $x^2 + 10x + 12 = 0$. Factoring this, $(x + 6)(x + 2) = 0$.

So either $x + 6 = 0$ or $x + 2 = 0$. Hence $x = -6$ or $x = -2$

Now, since we squared both sides to solve this equation, we **must** check our solutions.

If $x = -6$, then the left hand side of the original equation yields $-6 + 5 = -1$ while the right side yields $\sqrt{2(-6) + 13} = \sqrt{1} = 1$, so this solution does not check.

If $x = -2$, then the left hand side of the original equation yields $-2 + 5 = 3$ while the right side yields $\sqrt{2(-2) + 13} = \sqrt{9} = 3$, so this solution does check.

(l) $\sqrt{x + 8} = 2 + \sqrt{x}$

Solution: Squaring both sides: $x + 8 = (2 + \sqrt{x})$, or $x + 8 = 4 + 4\sqrt{x} + x$

Isolating the remaining radical term, $4 = 4\sqrt{x}$, or $1 = \sqrt{x}$.

Squaring again, $1 = x$.

Checking this solution: $\sqrt{1 + 8} = \sqrt{9} = 3$, while $2 + \sqrt{1} = 2 + 1 = 3$, so this solution is valid.

(m) $(y + 3)^{\frac{2}{3}} - 2(y + 3)^{\frac{1}{3}} - 3 = 0$

Solution: Here, we have an equation that is quadratic in form. By substituting $u = (y + 3)^{\frac{1}{3}}$, we obtain $u^2 - 2u - 3 = 0$, which factors as $(u - 3)(u + 1) = 0$.

Therefore, we have $u = 3$, or $u = -1$.

Going back to our substitution equation, if $(y + 3)^{\frac{1}{3}} = 3$, then, cubing both sides, $y + 3 = 27$, or $y = 24$.

On the other hand, if $(y + 3)^{\frac{1}{3}} = -1$, then, cubing both sides, $y + 3 = -1$, or $y = -4$.

Checking these, $(24 + 3)^{\frac{2}{3}} - 2(24 + 3)^{\frac{1}{3}} - 3 = (27)^{\frac{2}{3}} - 2(27)^{\frac{1}{3}} - 3 = 3^2 - 2(3) - 3 = 9 - 6 - 3 = 0$,

while $(-4 + 3)^{\frac{2}{3}} - 2(-4 + 3)^{\frac{1}{3}} - 3 = (-1)^{\frac{2}{3}} - 2(-1)^{\frac{1}{3}} - 3 == (-1)^2 - 2(-1) - 3 = 1 + 2 - 3 = 0$.

11. A movie theater charges adults \$7 per ticket wile children pay \$4 per ticket. At one afternoon showing, 50 total tickets sold for \$218. How many adults bought tickets to the show?

A careful reading of the situation described above suggests that this problem is best viewed as a “mixing problem”. We will let x be the number of tickets sold to adults and y the number of tickets sold to children.

From this, we obtain the equations $x + y = 50$ (50 total tickets are sold) and $7x + 4y = 218$ (a total \$218 was brought in from \$7 adult tickets and \$4 children’s tickets)

Since $x + y = 50$, then $y = 50 - x$, so we substitute to obtain: $7x + 4(50 - x) = 218$.

Then $7x + 200 - 4x = 218$, or $3x = 18$. Hence $x = \frac{18}{3} = 6$. (We then also know that $y = 50 - 6 = 44$.

Therefore, there were 6 adults who bought tickets to this show.