Math 127 - Exam 2 **Name:**____________________
Version 1
03/04/2015

You MUST show appropriate work to receive credit

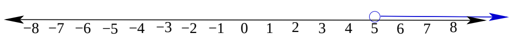
1. Solve the following inequalities. Express your answers in interval notation.

   (a) (5 points) $5x - (3x + 1) < 3(x - 2)$

   Simplifying, we get $5x-3x-1 < 3x-6$, or $2x-1 < 3x-6$

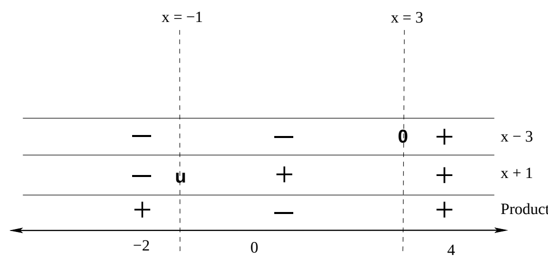
   Moving terms to isolate x, we get $-x < 5$, or $x > 5$.

   In interval notation, this is: $(5, \infty)$



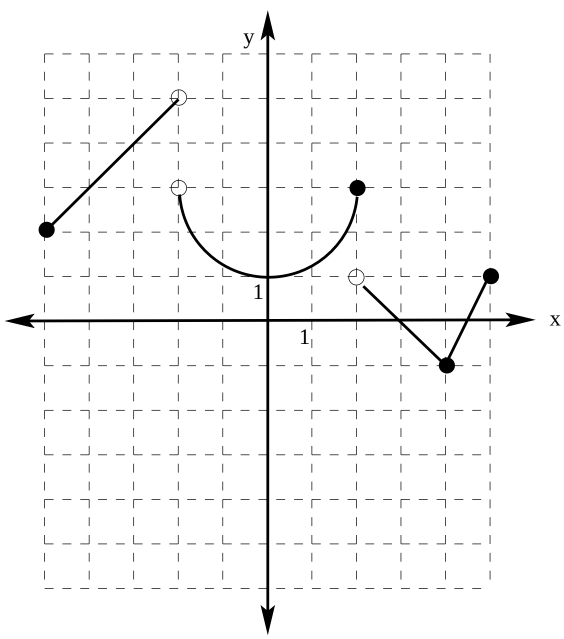
   (b) (7 points) $\dfrac{x-3}{x+1} \leq 0$

   Since this inequality has more than one term, to solve it, we must use either test points or a sign chart. Looking at the two factors, $x - 3$ and $x + 1$, the key values are $x = 3$ and $x = -1$. Using this, we construct the following sign chart:



   Notice that we want all negative values, the demoninator cannot be zero, but the numerator can be zero. Thus the solution set is given by $(-1, 3]$.

2. For the given graph of $f(x)$, find the following:



   (a) (2 points) The y-intercept(s). $(0, 1)$

   (b) (2 points) $f(4)$ $f(4) = -1$

   (c) (2 points) x, when $f(x) = 3$

   $x = -4$ and $x = 2$
   (note when $x = -2$ there is an open circle)

   (d) (2 points) The domain of f is $[-5, -2) \cup (-2, 5]$

   (e) (2 points) The range of f is $[-1, 5)$

   (f) (3 points) The intervals where f is increasing are $[-5, -2]$, $[0, 2]$, and $[4, 5]$.

3. Let $f(x) = x^2 - 3x$ and $g(x) = \sqrt{x+1}$ Find the following:

(a) (3 points) $f(a+1)$

$= (a+1)^2 - 3(a+1) = a^2 + 2a + 1 - 3a - 3$

$= a^2 - a - 2 = (a-2)(a+1)$

(b) (3 points) $(f-g)(2)$

$= f(2) - g(2) = (4-6) - \sqrt{2+1}$

$= -2 - \sqrt{3}.$

(c) (6 points) the domain of $\frac{g}{f}$ (in interval notation).

Notice that $\frac{f}{g}(x) = \frac{\sqrt{x+1}}{x^2-3x} = \frac{\sqrt{x+1}}{x(x-3)}$.

Then, in order for the output to be defined, we need two things: we must have $x + 1 \geq 0$, and we cannot have $x(x-3) = 0$.

Then $x \geq -1$, and we cannot have $x = 0$ or $x = 3$. Combining these, the domain is: $[-1, 0) \cup (0, 3) \cup (3, \infty)$

4. Given the points $A(3, 5)$ and $B(1, -3)$:

(a) (4 points) Find $d(A, B)$ (if necessary, round your answer to 2 decimal places)

$d(A, B) = \sqrt{(3-1))^2 + (5-(-3))^2} = \sqrt{2^2 + (8)^2} = \sqrt{4+64} = \sqrt{68} = 2\sqrt{17} \approx 8.25$

(b) (4 points) Find the midpoint of the line segment containing A and B.

$$M = \left(\frac{3+1}{2}, \frac{5-3}{2}\right) = \left(\frac{4}{2}, \frac{2}{2}\right) = (2, 1)$$

(c) (4 points) Find the equation for the line containing A and B in slope intercept form.

$$m = \frac{5-(-3)}{3-1} = \frac{8}{2} = 4.$$

Then, using point/slope, $y - 5 = 4(x-3)$, or $y - 5 = 4x - 12$.

Hence the equation for this line is: $y = 4x - 7$.

(d) (4 points) Find the equation for the line perpendicular to the line through A and B and containing the point $(-1, 2)$

First, since we are looking for a line that is perpendicular to the line found above, we see that $m = -\frac{1}{4}$.

Then, using point/slope and the point $(-1, 2)$, $y - 2 = -\frac{1}{4}(x+1)$, or $y - \frac{8}{4} = -\frac{1}{4}x - \frac{1}{4}$.

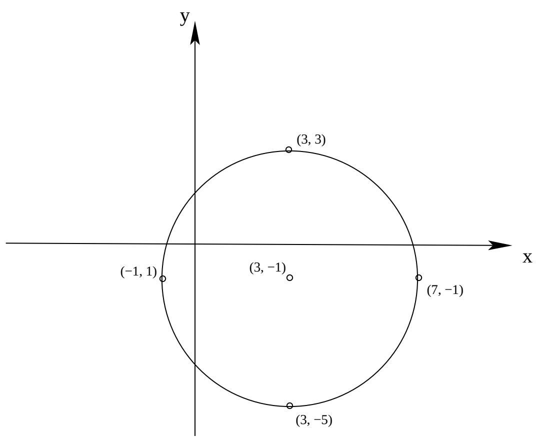
Hence the equation for this line is: $y = -\frac{1}{4}x - \frac{1}{4} + \frac{8}{4}$ or, simplifying, $y = -\frac{1}{4}x + \frac{7}{4}$.

5. (7 points) Find the center and radius of the circle with equation $x^2 + y^2 - 6x + 2y - 6 = 0$. Then graph the circle.

We begin by completing the square on each quadratic in the equation: $(x^2 - 6x + 9) + (y^2 + 2y + 1) = 6 + 9 + 1$

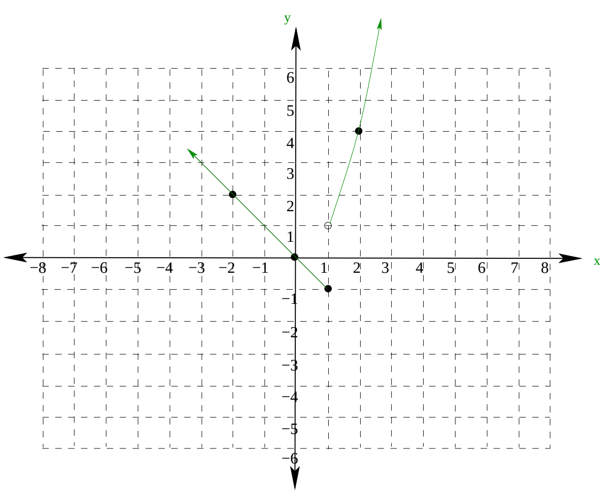
Hence the equation for this circle is: $(x-3)^2 + (y+1)^2 = 16$, so this circle has center $(3, -1)$ and radius $r = 4$.

Therefore, this circle has the following graph.



6. (6 points) Draw the graph of the function $f(x) = \begin{cases} -x & \text{if } x \leq 1 \\ x^2 & \text{if } x > 1 \end{cases}$.

Label your axes and at least 4 points on the graph.



x	$y = -x$
-2	2
-1	1
0	0
1	-1
x	$y = x^2$
1	1
2	4
3	9

7. Given that $f(x) = \dfrac{1}{x}$ and $g(x) = \sqrt{3x-2}$

(a) (3 points) Find $(f \circ g)(2)$

$$f(g(2)) = f\left(\sqrt{4}\right) = f(2) = \frac{1}{2}$$

(b) (3 points) Find $(g \circ f)(1)$

$$g(f(1)) = g(1) = \sqrt{3(1) - 2} = \sqrt{1} = 1.$$

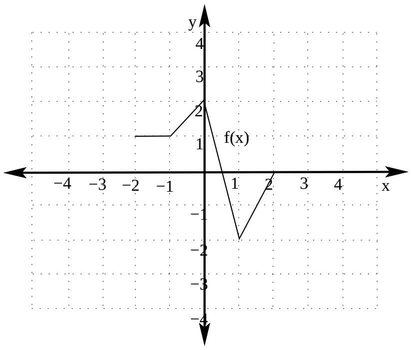
(c) (3 points) Find $(f \circ g)(x)$

$$f(g(x)) = f\left(\sqrt{3x-2}\right) = \frac{1}{\sqrt{3x-2}} = \frac{\sqrt{3x-2}}{3x-2}.$$

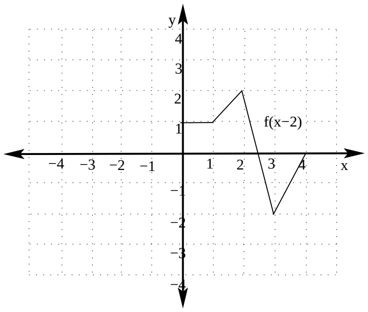
(d) (4 points) Find the domain of $(f \circ g)(x)$
To find the domain, we note that in order for $g(x)$ to be defined, we need $3x - 2 \geq 0$ and in order for $f(x)$ to be defined, it's demoninator must be non-zero, so we need $g(x) \neq 0$. Then the domain is found by solving the inequality $3x - 2 > 0$.

Then $3x > 2$, or $x > \frac{2}{3}$. Therefore, using interval notation, the domain of $(f \circ g)(x)$ is: $\left(\dfrac{2}{3}, \infty\right)$.

8. (5 points each) Given the graph of $f(x)$ shown below, use graph transformations to graph each of the following. Label at least 3 points on your final graphs.

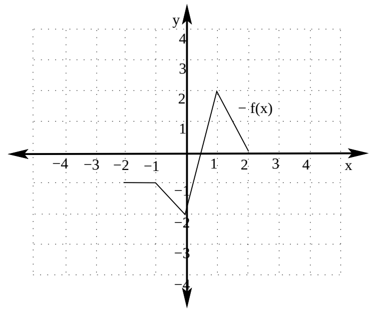


(a) $f(x-2)$



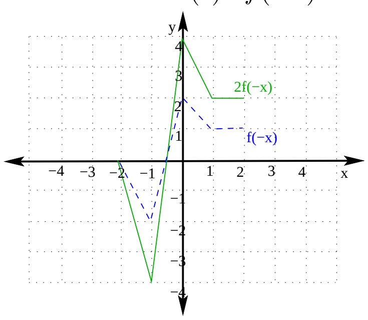
Shift Right 2 units

(b) $-f(x)$



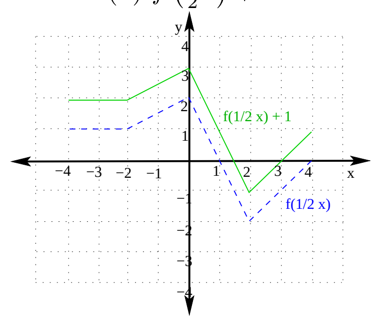
x - axis reflection

(c) $2f(-x)$



x - axis reflection, the vertically stretch by a factor of 2

(d) $f\left(\frac{1}{2}x\right)+1$



Horizontally stretch by a factor of 2, then up one unit.

9. In 1975, U.S. First Class postage cost \$0.10 per stamp. In 1985, the price of stamps had gone up to \$0.22

   (a) (4 points) Find a linear function that gives the cost of First Class postage as a function of time in years since 1975.

   Notice that x represents time in years since 1975. If we let y be the price of a first class stamp in cents (you could also choose dollars for your units as well, but using cents is a bit nicer), then we have the points $(0, 10)$ and $(10, 22)$.

   Then $m = \dfrac{22-10}{10-0} = \dfrac{12}{10} = \dfrac{6}{5} = 1.2$, and the y-intercept is $(0, 10)$.

   Then we have the line $y = 1.2x + 10$, so our linear model is $f(x) = 1.2x + 10$.

   (b) (4 points) Use your function to predict the cost of a First Class postage stamp today.

   First notice that today, 2015, corresponds to $x = 2015 - 1975 = 40$.

   Then, using out model, $f(40) = (1.2)(40) + 10 = 48 + 10 = 58$, so our model predicts that the cost of a first class stamp today is 58 cents.