Math 127
Exam 2 Practice Problem Solutions

1. Solve the following inequalities. Express your answer in interval notation.

   (a) $15 \leq -5x$
   Solution: Dividing by -5 and reversing the inequality, $-3 \geq x$.
   In interval notation this is: $(-\infty, -3]$

   (b) $.3x - .2(3x + 1) < 1$
   Solution: Distributing, $.3x - .6x - .2 < 1$, or $-.3x < 1.2$
   Multiplying by 10, $-3x < 12$. Then, dividing by -3, $x > -4$.
   In interval notation this is: $(-4, \infty)$

   (c) $-7 < 3x - 4 \leq 5$
   Solution: Adding 4 to each term, $-3 < 3x \leq 9$
   Then, dividing by 3, $-1 < x \leq 3$, which in interval notation this is: $(-1, 3]$

   (d) $2x - 5 \leq 5x - 2 \leq 2x + 7$
   Solution: Subtracting $2x$ from each part of the inequality, we get $-5 \leq 3x - 2 \leq 7$.
   Adding 2 to each part, we then have $-3 \leq 3x \leq 9$. Dividing by 3, we then have $-1 \leq x \leq 3$, which, in interval notation, is expressed $[-1, 3]$.

2. Determine whether or not the following equations are symmetric with respect to the x-axis, y-axis, or the origin.

   (a) $y = x^4 - x^2$
   Substituting $-x$ for x: $y = (-x)^4 - (-x)^2 = x^4 - x^2$. Thus the graph of this equation is symmetric with respect to the y-axis.
   Substituting $-y$ for y: $-y = x^4 - x^2$ or $y = -x^4 + x^2$, so the graph of this equation is *not* symmetric with respect to the x-axis.
   Finally, substituting $-x$ for x *and* $-y$ for y: $-y = (-x)^4 - (-x)^2$, or $y = -x^4 + x^2$ so the graph of this equation is *not* symmetric with respect to the origin.

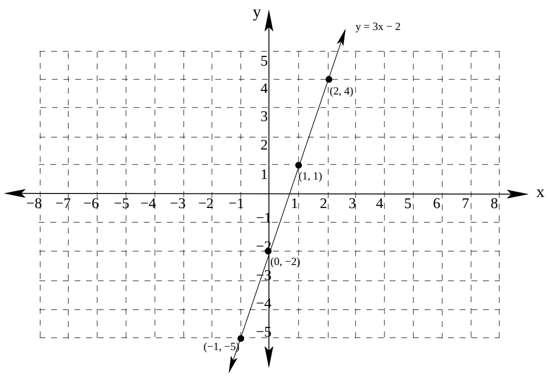
   (b) $y = x^3 - 2x$
   Substituting $-x$ for x: $y = (-x)^3 - 2(-x) = -x^3 + 2x$, so the graph of this equation is *not* symmetric with respect to the y-axis.
   Substituting $-y$ for y: $-y = x^3 - 2x$, or $y = -x^3 + 2x$, so the graph of this equation is *not* symmetric with respect to the y-axis.
   However, substituting $-x$ for x *and* $-y$ for y: $-y = (-x)^2 - 2(-x) = -x^3 + 2x$, or, dividing both sides by -1, $y = x^3 - 2x$ so the graph of this equation is symmetric with respect to the origin.

   (c) $x^2 - y^2 = 1$
   Here, substituting either $-x$ for x or $-y$ for y or substituting both $-x$ for x *and* $-y$ for y gives back $x^2 - y^2 = 1$ when we simplify, so this equation is symmetric with respect to the y-axis, the x-axis, and with respect to the origin.

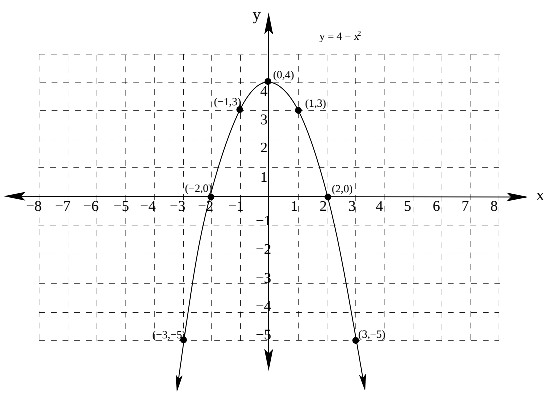
   (d) $y = 3x - 2$
   Substituting $-x$ for x gives: $y = -3x - 2$
   Substituting $-y$ for y gives: $-y = 3x - 2$ or $y = -3x + 2$
   Substituting $-x$ for x *and* $-y$ for y gives: $-y = -3x - 2$, or $y = 3x + 2$.
   Therefore, this equation is *neither* symmetric with respect to the y-axis, nor the x-axis nor with respect to the origin.

3. Sketch the graphs of the following functions. Be sure to find and label all x and y intercepts.

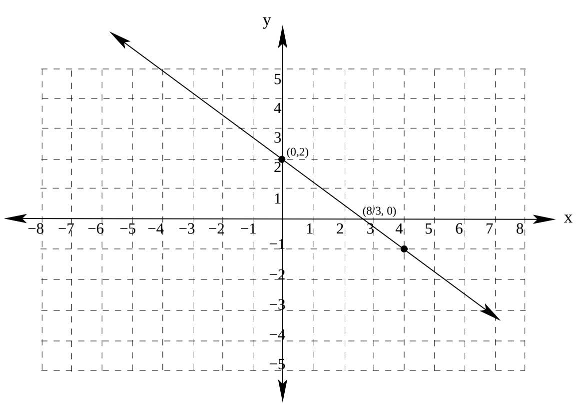
(a) $y = 3x - 2$



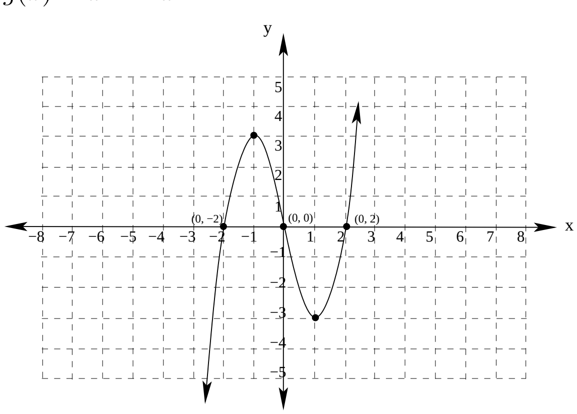
(b) $y = 4 - x^2$



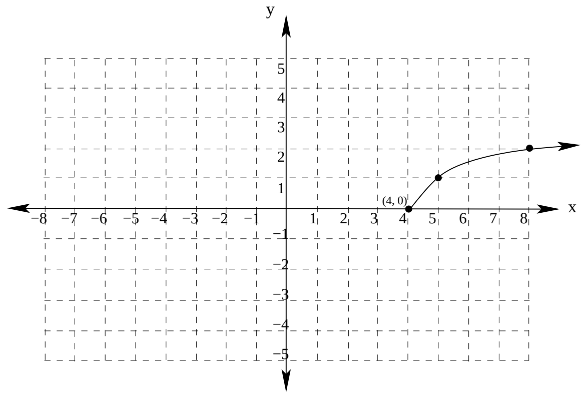
(c) $f(x) = -\frac{3}{4}x + 2$



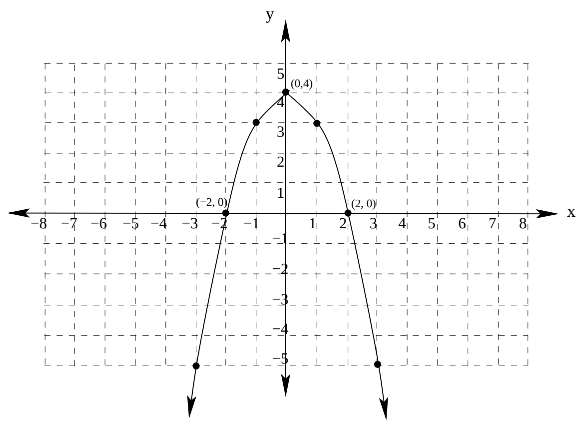
(d) $g(x) = x^3 - 4x$



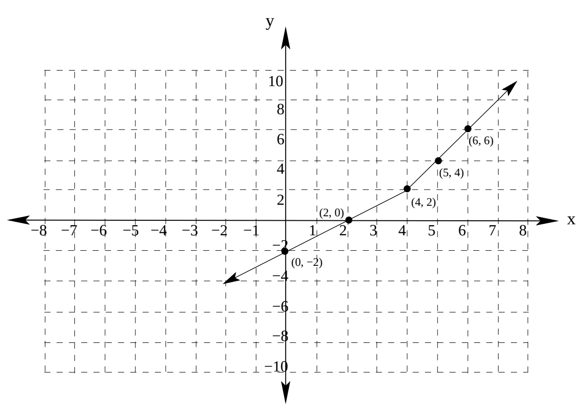
(e) $y = \sqrt{x-4}$



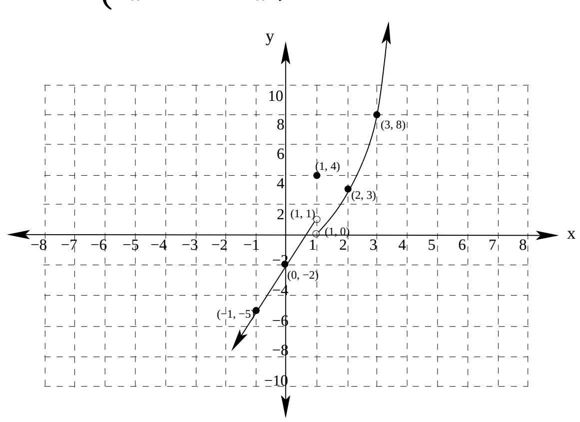
(f) $y = 4 - x^2$



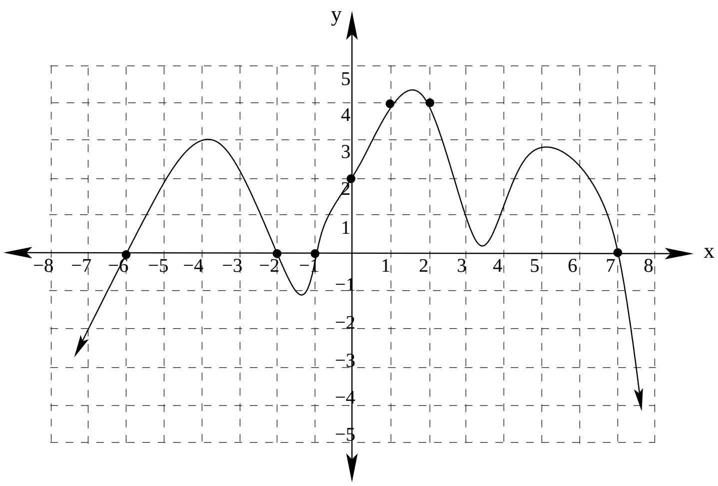
(g) $f(x) = \begin{cases} x-2 & \text{if } x \le 4 \\ 2x-6 & \text{if } x > 4 \end{cases}$



(h) $f(x) = \begin{cases} 3x-2 & \text{if } x < 1 \\ 4 & \text{if } x = 1 \\ x^2-1 & \text{if } x > 1 \end{cases}$



4. Based on the graph given below:

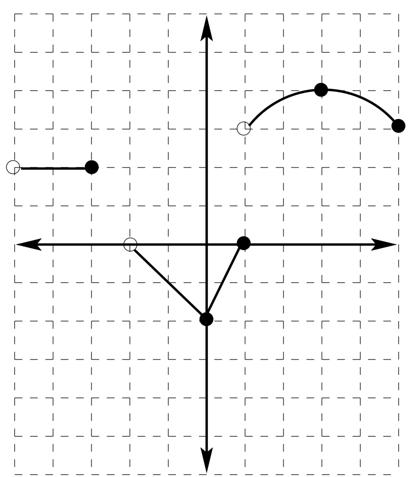


(a) Find the coordinates of all x intercepts.
From the graph, we see that the x-intercepts are at the points: $(-6, 0), (-2, 0), (-1, 0)$ and $(7, 0)$

(b) Find the coordinates of all y intercepts.
From the graph, we see that the y-intercept is at the point: $(0, 2)$

(c) Find the x-value(s) when $y = 4$
From the graph, we see that the x-values corresponding to $y = 4$ are $x = 1$ and $x = 2$.

5. For the given graph of $f(x)$, find the following:



(a) $f(0) = -2$

(b) $f(3) = 4$

(c) x, when $f(x) = 2$
$f(x) = 2$ for every x in the interval $(-5, 3]$

(d) The domain of f
The domain of f is $(-5, -3] \cup (-2, 5]$

(e) The range of f
The range of f is $[-2, 0] \cup \{2\} \cup [3, 4]$

(f) The intervals where f is decreasing.
f is decreasing on $(-2, 0] \cup [3, 5]$

6. Let $f(x) = x^2 - 2x$. Find and simplify the following:

(a) $f(2)$, and $f(\frac{2}{3})$
$f(2) = 2^2 - 2(2) = 4 - 4 = 0$ and $f(\frac{2}{3}) = (\frac{2}{3})^2 - 2(\frac{2}{3}) = \frac{4}{9} - \frac{4}{3} = \frac{4}{9} - \frac{12}{9} = -\frac{8}{9}$

(b) $f(a + 3)$
$f(a + 3) = (a + 3)^2 - 2(a + 3) = a^2 + 6a + 9 - 2a - 6 = a^2 + 4a + 3 = (a + 1)(a + 3)$

(c) $f(2a - 1)$
$f(2a - 1) = (2a - 1)^2 - 2(2a - 1) = 4a^2 - 4a + 1 - 4a + 2 = 4a^2 - 8a + 3 = (2a - 1)(2a - 3)$

(d) $\dfrac{f(a + h) - f(a)}{h}$

$$\frac{f(a + h) - f(a)}{h} = \frac{(a + h)^2 - 2(a + h) - (a^2 - 2a)}{h} = \frac{a^2 + 2ah + h^2 - 2a - 2h - a^2 + 2a}{h}$$
$$= \frac{2ah + h^2 - 2h}{h} = \frac{h(2a + h - 2)}{h} = 2a + h - 2$$

7. Determine whether or not the following are functions:

(a) $\{(3, 4), (5, 7), (2, -1), (6, 8), (8, 6)\}$
Since there are no repeated x-coordinates in this set or ordered pairs, this is a function.

(b) $\{(1,2),(3,7),(4,-12),(5,8),(7,2)\}$
Since there are no repeated x-coordinates in this set or ordered pairs, this is a function.

(c) $\{(1,2),(2,3),(3,4),(4,5),(3,5)\}$
Since there are two ordered pairs with 3 as their x-coordinate, this is **not** a function.

8. Given the points $A(2,-2)$ and $B(-1,4)$:

(a) Find $d(A,B)$
$d(A,B)=\sqrt{(2-(-1))^2+(-2-4)^2}=\sqrt{3^2+(-6)^2}=\sqrt{9+36}=\sqrt{45}=3\sqrt{5}$.

(b) Find the midpoint of the line segment containing A and B.
$$M=\left(\frac{2-1}{2},\frac{-2+4}{2}\right)=\left(\frac{1}{2},\frac{2}{2}\right)=\left(\frac{1}{2},1\right)$$

(c) Find the equation for the line containing A and B in general form.
$m=\frac{4-(-2)}{-1-2}=\frac{6}{-3}=-2$, so, using the point/slope equation:
$y+2=-2(x-2)=-2x+4$
Thus the line has equation $y=-2x+2$.

(d) Find the equation for the circle centered at B containing the point A.
From part (a) above, $r=3\sqrt{5}$ and $C=(-1,4)$.
Therefore, the circle has equation $(x+1)^2+(y-4)^2=45$

(e) Find an equation for the vertical line containing B.
$x=-1$

(f) Find an equation for the horizontal line containing A.
$y=-2$

9. Find the equation for each line described below. Put your final answer in slope/intercept form.

(a) The line containing the points $(-4,1)$ and $(3,-7)$
First, we find the slope of this line: $m=\dfrac{1-(-7)}{-4-3}=-\dfrac{8}{7}$
Then, we use the point/slope formula: $y-1=-\frac{8}{7}(x+4)$ or $y=-\frac{8}{7}x-\frac{32}{7}+1$
Thus $y=-\frac{8}{7}x-\frac{25}{7}$

(b) The line parallel to the line $3x-4y=12$ passing through the point $(1,3)$
Putting this line into slope intercept form, we have: $4y=3x-12$, or $y=\frac{3}{4}x-3$
Then, since we are looking for a parallel line, we need a line with slope $m=\frac{3}{4}$ passing through $(1,3)$.
Then, using the point/slope formula: $y-3=\frac{3}{4}(x-1)$ or $y=\frac{3}{4}x-\frac{3}{4}+3$
Thus $y=\frac{3}{4}x+\frac{9}{4}$

(c) The line perpendicular to the line $5y-2x=3$ and having x-intercept -1.
Putting this line into slope intercept form, we have: $5y=2x+3$, or $y=\frac{2}{5}x+\frac{3}{5}$
Then, since we are looking for a perpendicular line, we need a line with slope $m=-\frac{5}{2}$ passing through $(-1,0)$, since the x-intercept is -1.
Then, using the point/slope formula: $y-0=-\frac{5}{2}(x+1)$ or $y=-\frac{5}{2}x-\frac{5}{2}$

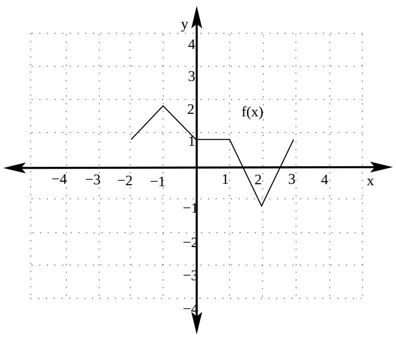
10. A 16oz jar of peanut butter cost \$1.78 in 1995. In 2005, a similar jar cost \$2.99.

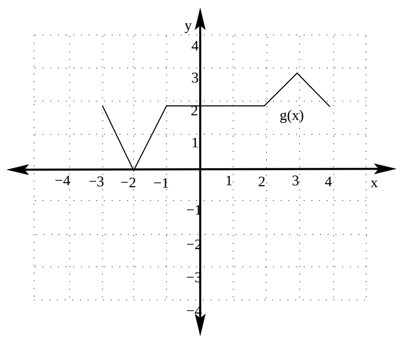
(a) Find a line that models the price of peanut butter over time (hint: you can take $x=0$ to represent 1995)
Using the points $(0,1.78)$ and $(10,2.99)$, we find $m=\frac{2.99-1.78}{10-0}=.121$ and $b=1.78$.
Therefore, the line modeling the price of peanut butter is given by: $y=.121x+1.78$, where $x=0$ corresponds to the year 1995.

(b) Use your model to predict the price of peanut butter in 2010.
2010 corresponds to $x=2010-1995=15$, and so $y=.121(15)+1.78=\$3.595$, or around \$3.60.

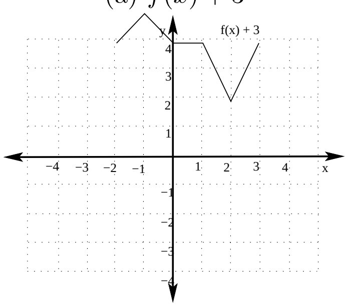
(c) According to your model, when will the price of peanut butter reach \$5.00 for a 16oz jar?
If $y=\$5.00$, then $5=.121x+1.78$, so $5-1.78=.121x$, or $3.22=.121x$
Therefore, $x=\frac{3.22}{.121}=26.61$.
Hence, according to this model, the price of peanut butter will reach \$5 per 16 oz jar 26.61 years after 1995, or sometime during 2022.

11. Given the graphs of $f(x)$ and $g(x)$ shown below, use graph transformations to graph each of the following. Label at least 3 points in your final graph.

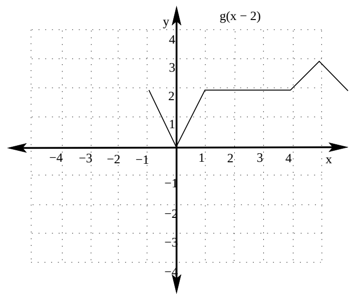




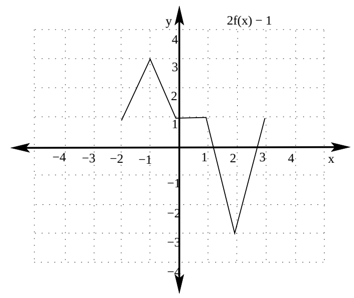
(a) $f(x) + 3$



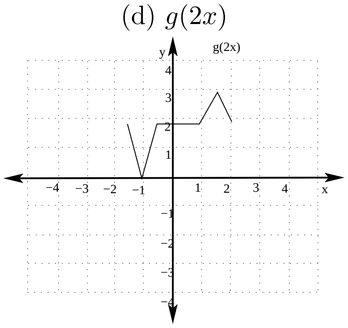
(b) $g(x - 2)$



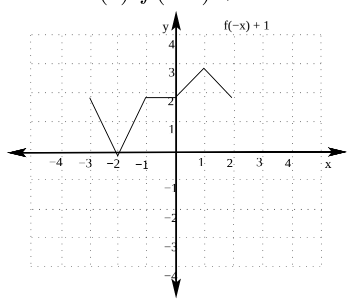
(c) $2f(x) - 1$



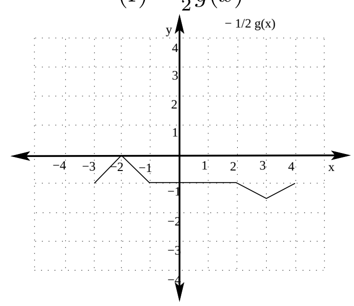
(d) $g(2x)$



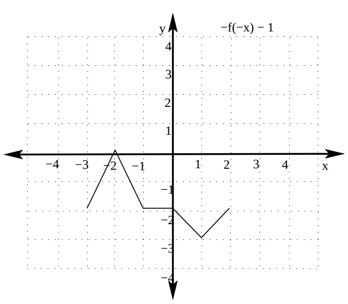
(e) $f(-x) + 1$



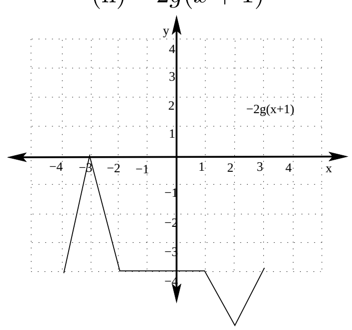
(f) $-\frac{1}{2}g(x)$



(g)c $-f(-x) - 1$



(h) $-2g(x + 1)$

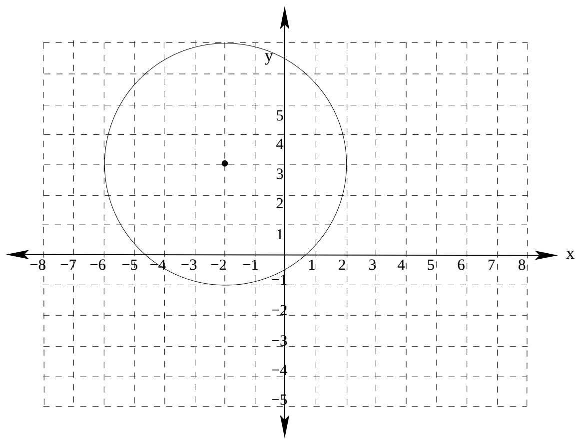


12. Find the equation for the following circles:

    (a) The circle with center $(4, -5)$ and radius 6
    The circle has equation $(x - 4)^2 + (y + 5)^2 = 36$

    (b) The circle with center $(2, 1)$ and passing through the point $(5, 5)$
    Notice that the distance between these point is: $d(A, B) = \sqrt{(5-2)^2 + (5-1)^2} = \sqrt{3^2 + 4^2} = \sqrt{9 + 16} = \sqrt{25} = 5$.
    Therefore, $r = 5$ and $C = (2, 1)$, so the circle has equation $(x - 2)^2 + (y - 1)^2 = 25$

13. Graph the circle with equation $x^2 + y^2 + 4x - 6y - 3 = 0$
    Rearranging the terms and completing the square: $x^2 + 4x + \quad + y^2 - 6y + \quad = 3$
    Therefore, $x^2 + 4x + 4 + y^2 - 6y + 9 = 3 + 4 + 9$, or $(x + 2)^2 + (y - 3)^2 = 16$
    Thus this circle has center $(-2, 3)$ and radius $r = 4$, so the graph of the circle is:



14. Find the domain of the following functions (put your answers in interval notation):

(a) $f(x) = \frac{x^2+x-2}{x^2-4}$
We need to avoid making the denominator zero, so we can't have $x^2 - 4 = 0$ or $x^2 = 4$.
Therefore, $x \neq \pm 2$.
Therefore, in interval notation, the domain of f is: $(-\infty, -2) \cup (-2, 2) \cup (2, \infty)$.

(b) $f(x) = \frac{\sqrt{4-2x}}{x^2-1}$
There are two things to worry about. First, we need the denominator to be non-zero. That is, we can't have $x^2 - 1 = 0$ or $x^2 = 1$.
Therefore, $x \neq \pm 1$.
Next, we can't take the square root of a negative number, so we need $4 - 2x \geq 0$.
That is, $4 \geq 2x$, or $2 \geq x$. Combining these, the domain of f is:
$(-\infty, -1) \cup (-1, 1) \cup (1, 2]$

(c) $f(x) = \frac{4}{\sqrt{3x-5}}$
Here, we need $3x - 5 > 0$, or $3x > 5$. Thus $x > \frac{5}{3}$.
Therefore, the domain is: $(\frac{5}{3}, \infty)$

(d) $f(x) = \frac{\sqrt{3-2x}}{2x^2+x-15}$
There are two things to worry about. First, we need the denominator to be non-zero. That is, we can't have $2x^2 + x - 15 = 0$ or $(2x-5)(x+3) = 0$.
Therefore, $x \neq \frac{5}{2}$ or $x \neq -3$.
Next, we can't take the square root of a negative number, so we need $3 - 2x \geq 0$.
That is, $3 \geq 2x$, or $\frac{3}{2} \geq x$. Combining these, the domain of f is:
$(-\infty, -3) \cup (-3, \frac{3}{2}]$

15. Given that $f(x) = \sqrt{3x-2}$ and $g(x) = x^2 - 4$

(a) Find $\dfrac{g}{f}(3)$
$f(3) = \sqrt{3(3) - 2} = \sqrt{7}$
$g(3) = 3^2 - 4 = 9 - 4 = 5$
Therefore, $\dfrac{g}{f}(3) = \dfrac{g(3)}{f(3)} = \dfrac{5}{\sqrt{7}} = \dfrac{5\sqrt{7}}{7}$

(b) Find $f \circ g(2)$
$g(2) = 4 - 4 = 0$
$f \circ g(2) = f(g(2)) = f(0) = \sqrt{3(0) - 2} = \sqrt{-2}$ which is not a real number.

(c) Find $g \circ f(x)$
$g \circ f(x) = g(f(x)) = (\sqrt{3x-2})^2 - 4 = 3x - 2 - 4 = 3x - 6 = 3(x-2)$

(d) Find $f \circ g(x)$
$f \circ g(x) = f(g(x)) = \sqrt{3(x^2-4) - 2} = \sqrt{3x^2 - 12 - 2} = \sqrt{3x^2 - 14}$

(e) Find the domain of $g \circ f(x)$. Give your answer in interval notation.
To find the domain of $g \circ f(x) = g(f(x))$, we first find the domain of f:
$3x - 2 \geq 0$, so $3x \geq 2$ or $x \geq \frac{2}{3}$.
Next, notice that g is never undefined.
Therefore, the domain of $g \circ f(x)$ is $[\frac{2}{3}, \infty)$

(f) Find the domain of $\dfrac{f}{g}$. Give your answer in interval notation.
To be in the domain of $\dfrac{f}{g}$, we need $f(x)$ to be defined, and $g(x)$ to be defined and non-zero.
Therefore, we need $3x - 2 \geq 0$, or $3x \geq 2$, hence $x \neq \frac{2}{3}$.
We also need $x^2 - 4 \neq 0$, or $x \neq \pm 2$
Hence the domain of $\dfrac{f}{g}$ is $[\frac{2}{3}, 2) \cup (2, \infty)$

16. An oil well off the Gulf Coast is leaking, with the leak spreading oil over the surface in the shape of a circle. At any time t, in minutes, after the beginning of the leak, the radius of the circular oil slick on the surface is $r(t) = 4t$ feet. Let $A(r) = \pi r^2$ represent the area of the circle of radius r.

(a) Find $(A \circ r)(t)$
Since $r(t) = 4t$ and $A(r) = \pi r^2$, $(A \circ r)(t) = \pi(4t)^2 = 16\pi t^2$

(b) Explain what $(A \circ r)(t)$ is in practical terms.
$(A \circ r)(t)$ gives the area of the oil as a function of time in minutes.