Math 127 - Exam 3 **Name:**______________________________
Version 1
04/10/2015

You MUST show appropriate work to receive credit

1. Use properties of exponents and radicals to simplify the following expression. Your answer should have no negative exponents. Assume all variables represent nonnegative numbers.

   (a) (5 points) $\left(\dfrac{10x^{-2}y^3}{2^3x^5y^{-5}}\right)^2$

   $$\left(\frac{10x^{-2}y^3}{2^3x^5y^{-5}}\right)^2 = \left(\frac{10\cdot y^3y^5}{8x^5\cdot x^2}\right)^2 = \left(\frac{5y^8}{4x^7}\right)^2 = \frac{25y^{16}}{16x^{14}}.$$

   (b) (5 points) $\sqrt[3]{8x^6y^7z^8}$

   $$= \sqrt[3]{2^3x^6y^6\cdot y\cdot z^6\cdot z^2} = 2x^2y^2z^2\sqrt[3]{yz^2}.$$

2. (3 points each) Given the tables below, find the following:

x	$f(x)$	$g(x)$
0	-4	1
1	2	-3
2	1	4
3	7	2
4	1	0

   (a) $f^{-1}(2) = 1$ [since $f(1) = 2$]

   (b) $f(g^{-1}(4))$

   Since $g^{-1}(4) = 2$, then $f(g^{-1}(4)) = f(2) = 1$

3. (7 points) Find the inverse of the function $f(x) = \dfrac{3x-1}{2x+5}$. You **do not** need to compute compositions to verify your result.

   $y = \dfrac{3x-1}{2x+5}$

   Multiplying both sides by $2x+5$ gives: $y(2x+5) = 3x-1$

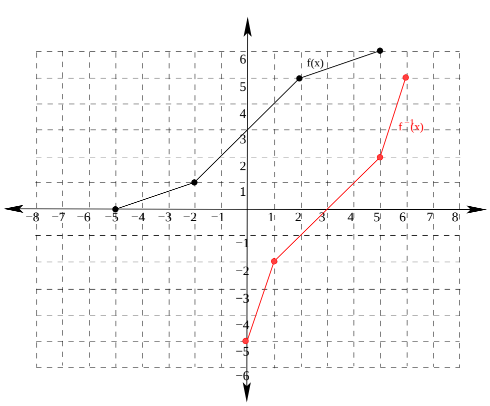
   Then $2xy + 5y = 3x - 1$ or $5y + 1 = 3x - 2xy$

   Hence $5y + 1 = x(3-2y)$

   so $x = \dfrac{5y+1}{3-2y}$

   Therefore, $f^{-1}(x) = \dfrac{5x+1}{3-2x}$

4. Given the following graph of $f(x)$:



   (a) (4 points)Draw the graph of $f^{-1}(x)$ on the axes provided.

   See graph.

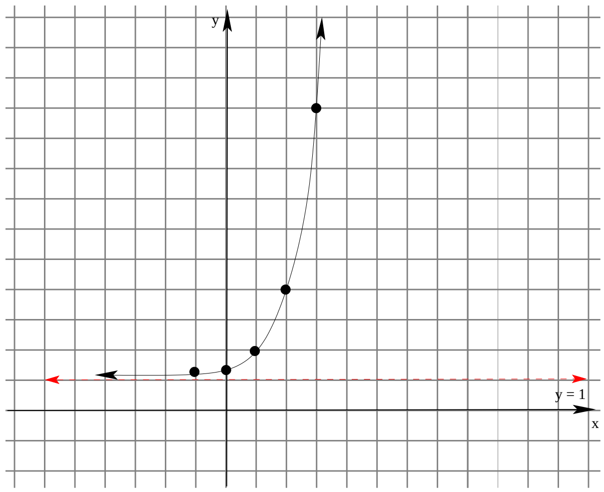
   (b) (4 points) Give the domain and range of $f^{-1}(x)$

   Notice that the Domain of $f(x)$ is $[-5, 5]$ and the Range of $f(x)$ is $[0, 6]$.

   Therefore, the Domain of $f^{-1}(x)$ is $[0, 6]$ and the Range of $f^{-1}(x)$ is $[-5, 5]$.

   (We can also see this directly from the graph of $f^{-1}(x)$)

5. (7 points) Graph the function $f(x) = 3^{x-1} + 1$. Graph and label the asymptote and at least 3 points on your graph. Also give the domain and range.



x	3^{x+1}	$f(x) = 3^{x-1} + 1$
-1	$\frac{1}{9} + 1 = \frac{10}{9}$	$-\frac{8}{9}$
0	$3^{-1} + 1 = \frac{4}{3}$	$-\frac{2}{3}$
1	$3^0 + 1 = 2$	0
2	$3^1 + 1 = 4$	2
3	$3^2 + 1 = 10$	8

Domain: $(-\infty, \infty)$

Range: $(1, \infty)$

Asymptote:
the horizontal line $y = 1$

6. (5 points) Suppose you have \$7,000 to invest. Find the amount you would have after 12 years if you deposit your \$7,000 in an account that pays 5% annual interest compounded monthly.

Recall that $A = P\left(1 + \frac{r}{n}\right)^{nt}$. Here, $A = \$7000$, $r = 0.05$, $n = 12$, and $t = 12$.

Then we have $A = 7,000\left(1 + \frac{0.05}{12}\right)^{(12)(12)} \approx \$12,738.94$

7. (2 points each) Find the *exact value* of each of the following:

(a) $\log_9(81) = 2$

Since $9^2 = 81$

(b) $\log_{13}(1) = 0$

Since $13^0 = 1$

(c) $\log_\pi(\sqrt[3]{\pi}) = \log_\pi\left(\pi^{\frac{1}{3}}\right) = \frac{1}{3}$

(d) $21^{\log_{21}(7)} = 7$

By the inverse function composition property.

8. (5 points) Use properties of logarithms to **expand** the expression: $\ln\left(\frac{(x+2)^3}{e^4 z^2}\right)$

$= \ln\left(\frac{(x+2)^3}{e^4 z^2}\right) = \ln(x+2)^3 - \ln e^4 z^2$

$== 3\ln(x+2) - \left[\ln(e^4) + \ln(z^2)\right] = 3\ln(x+2) - 4\ln(e) - 2\ln z = 3\ln(x+2) - 4 - 2\ln z$

9. (4 points) Use the change of base formula to compute $\log_{13}(500)$ to 4 decimal places.

$$\log_{13}(500) = \frac{\ln 500}{\ln 13} \approx 2.4229$$

10. (7 points each) Solve the following equations. Give an exact expression for your solution first (and an approximationto three decimal places, if appropriate)

(a) $2^{3-2x} = 8^{x+2}$

$2^{3-2x} = \left(2^3\right)^{x+2}$

So $3 - 2x = 3(x+2)$ or $3 - 2x = 3x + 6$.

Then $-3 = 5x$, thus $x = -\frac{3}{5}$

(b) $5e^{2x} = 15$

We begin by dividing both sides by 5: $e^{2x} = 3$

Then $\ln\left(e^{2x}\right) = \ln(3)$

Then $2x = \ln 3$, so $x = \frac{\ln 3}{2} \approx 0.549$

(c) $\log(x) + \log(x-3) = 1$

We begin by combining the left hand side into a single logarithmic expression:

$\log\left(x(x-3)\right) = 1$

Then, rewriting in exponential form: $10^1 = x(x-3)$

Then $10 = x^2 - 3x$ or $x^2 - 3x - 10 = 0$

Factoring this gives $(x-5)(x+2) = 0$

So there are two potential solutions: $x = 5$ and $x = -2$.

Check: (we must check logarithmic equations)

Notice that when $x = -2$ $\log(-2)$ and $\log(-2-3) = \log(-5)$ are undefined, so we reject this solution.

When $x = 5$, $\log(5) + \log(5-3) = \log(5) + \log(2) = \log(5 \cdot 2) = \log 10 = 1$ ✓

Thus there is one solution: $x = 5$.

(d) $2^{2x+1} = 5^{3x}$

Taking the logarithm of each side: $\ln 2^{2x+1} = \ln 5^{3x}$

Then $(2x+1)\ln 2 = 3x \cdot \ln 5$, or, distributing, $(2\ln 2)x + \ln 2 = (3\ln 5)x$

Moving terms, we have $\ln 2 = (3\ln 5)x - (2\ln 2)x$

Therefore, $\ln 2 = x(3\ln 5 - 2\ln 2)$

Thus $x = \dfrac{\ln 2}{3\ln 5 - 2\ln 2} \approx 0.201$

11. Suppose that the population of Algebronia is growing according to the model $f(t) = 15e^{0.034t}$ where t is in years since 2000, and $f(t)$ is in millions of people.

(a) (3 points) Find the population of Algebronia in the year 2000.

Notice that the year 2000 corresponds to $t = 0$. so the population in 2000 is given by $f(0) = 15e^{0.034(0)} = 15e^0 = 15$, or 15 million people.

Therefore, there were 15 million people living in Algebronia in the year 2000.

(b) (3 points) Find the populations of Algebronia today (to the nearest person).

Notice that today corresponds to $t = 2015 - 2000 = 15$, so the population today is given by $f(13) = 15e^{0.034(15)} \approx 24.979368$

So there are 24,979,368 people living in Algebronia today (notice that you were asked to find the population to the *nearest person*).

(c) (5 points) Find the year that the population of Algebronia reaches 40 million people.

To find the year in which the population will reach 40 million, we solve the equation $40 = 15e^{0.034t}$

Then $\frac{40}{15} = e^{0.034t}$, or $\ln\left(\frac{40}{15}\right) = 0.034t$

Hence $t = \dfrac{\ln\left(\frac{40}{15}\right)}{0.034} \approx 28.85$.

Therefore, the population of Algebronia will reach 40 million sometime during the year 2029.

12. (7 points) Suppose that laboratory research has shown that a 12 gram sample of a newly discovered substance called Wonderflonium is reduced down to 7 grams in 5 hours. Find the half life of Wonderflonium.

We will use the continuous decay model: $A = A_0e^{kt}$. To find the half life, we must first use the information we know about Wonderflonium to find the growth constant k. In the lab test, we had $A_0 = 12$ grams, $A = 7$ grams, and $t = 5$ hours.

Then $7 = 12e^{5k}$, so $\frac{7}{12} = e^{5k}$, or $\ln\frac{7}{12} = 5k$. Thus $k = \frac{\ln\frac{7}{12}}{5} \approx -0.1078$.

Using this, we set up the half-life equation: $\frac{1}{2} \approx e^{-0.1078t}$.

Solving this for t, we have $\ln\frac{1}{2} \approx -0.1078t$, so $t \approx \frac{\ln\frac{1}{2}}{-0.1078} \approx 6.43$

Hence the half-life of Wonderflonium is about 6.43 hours.