Math 127
Polynomial Practice Problems

1. For each quadratic function given below, find the coordinates of the vertex, and find the equation for the line of symmetry for the graph of the function.

   (a) $f(x) = (x-5)^2 + 12$
   (b) $f(x) = (x+3)^2 - 8$
   (c) $f(x) = 2x^2 - 12x + 22$
   (d) $f(x) = -4x^2 + 16x - 13$

2. For each quadratic function given below, find the vertex, line of symmetry, and (if applicable) the x-intercepts of the function. Then graph the function carefully in the xy-plane.

   (a) $f(x) = x^2 - 4x$
   (b) $f(x) = 6x^2 + 7x - 24$
   (c) $f(x) = -2x^2 + 20x - 43$

3. For each polynomial function given below, use the degree of the polynomial and the sign of the leading coefficient to describe the end behavior of the polynomial.

   (a) $f(x) = 5x^4 - 3x^2 + 7$
   (b) $f(x) = -2x^3 + 4x^2 - 15x + 12$
   (c) $f(x) = 12x^5 - 5x^4 + 3x^2 - 12x + 7$
   (d) $f(x) = -7x^6 + 5x^4 - 13x^3 + 21x^2 + 15x + 23$

4. For each polynomial function given below, find the zeros of the polynomial and also give the multiplicity of each zero.

   (a) $f(x) = x^4 + 3x^3 - 4x^2$
   (b) $f(x) = x^3 + 2x^2 - 4x - 8$
   (c) $f(x) = x(x-4)^4(x+3)^3$
   (d) $f(x) = x^3(x-2)^2(x^2-4)^5$

5. Use the intermediate value theorem to show that $f(x) = 2x^3 + 5x^2 - 3$ has a zero between $x = -3$ and $x = -2$

6. For each of the following quadratic functions:
   - Find the vertex and axis of symmetry
   - Find all intercepts
   - Graph the function and list its range in interval notation

   (a) $f(x) = 4(x+3)^2 - 7$
   (b) $f(x) = -3(x-4)^2 + 11$
   (c) $f(x) = x^2 - 2x - 15$
   (d) $f(x) = 3x^2 - 2x - 4$
   (e) $f(x) = -2x^2 + 8x - 1$

7. Among all pairs of numbers whose sum is 20, find the pair whose product is as large as possible. Also find the value of the maximum product.

8. Suppose that you have 600 feet of fencing to enclose a rectangular field that borders a river on one side. If you do not fence the side along the river, find the length and width of the maximum sized field that you can fence in.

9. Find the zeros for each polynomial function given. Also find the multiplicity of each zero.

   (a) $f(x) = x^3 - x^2 - 9x + 9$
   (b) $f(x) = 2x^3 - 5x^2 - 12x$
   (c) $f(x) = x^2(x-1)(x+3)^3$
   (d) $f(x) = x^3(x-2)(x^2-4)^2$

10. Use the Leading Term Test to determine the end behavior for each polynomial function given.

    (a) $f(x) = x^5 - 4x^3 + 17x^2 + 5x - 14$
    (b) $f(x) = -3x^8 - 5x^5 + 3x^2 + 17$
    (c) $f(x) = x^2(2x - 1)(x + 2)$
    (d) $f(x) = -4x^2(x - 2)(x^2 + 1)$

11. Use the Intermediate Value Theorem to show that each polynomial has a zero on the given interval.

    (a) $f(x) = x^4 - 3x^2 + 5x - 7$ on $[1, 2]$
    (b) $f(x) = 4x^3 - 12x^2 + 25x - 100$ on $[3, 5]$

12. Use the Leading Coefficient test to determine the end behavior, find all x-intercepts and their multiplicities, find the y-intercept and any symmetry, and then graph the given polynomial.

    (a) $f(x) = 2x^3 + 3x^2 - 2x$
    (b) $f(x) = x^3 - x^2 - 16x + 16$
    (c) $f(x) = x^2(x - 1)(x + 4)$
    (d) $f(x) = x^3(x + 1)(x - 2)^2(x - 4)$

13. Use Long Division to find the following Quotients:

    (a) $\dfrac{4x^4 - 4x^2 + 6x}{x - 4}$
    (b) $\dfrac{x^4 - 3x^2 + 7x + 5}{x^2 + 1}$
    (c) $\dfrac{3x^5 - 7x^3 + 5x^2 - 3}{x^2 - x + 3}$

14. Use synthetic division to find the indicated function values:

    (a) $f(7)$ if $f(x) = x^3 - 4x^2 + 7x - 5$
    (b) $f(-1)$ if $f(x) = x^5 - 3x^3 + 3x + 4$
    (c) $f(2)$ if $f(x) = x^7 - 3x^3 + 4x - 1$

15. Solve the following equations:

    (a) $2x^3 - 3x^2 - 11x + 6 = 0$ given that -2 is a zero of $f(x) = 2x^3 - 3x^2 - 11x + 6$
    (b) $3x^3 + 7x^2 - 22x - 8 = 0$ given that $-\frac{1}{3}$ is a zero of $f(x) = 3x^3 + 7x^2 - 22x - 8$

16. Use the Rational Zero Theorem to find all possible zeros for each polynomial given.

    (a) $f(x) = x^3 + 3x^2 - 6x - 8$
    (b) $f(x) = 2x^4 + 3x^3 - 11x^2 - 9x + 15$
    (c) $f(x) = 4x^5 - 8x^4 - x + 2$

17. Find a 5th degree polynomial $f(x)$ for which $\frac{1}{2}$, -1, 2, and $\pm i$ are all zeros and with $f(1) = 12$.