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Mathematics of the Human-Vampire Conflict

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Do vampires exist? I'll bet most of you would say "Of course not!," but how can you really be sure? In a recent article, "Ghosts, Vampires and Zombies: Cinema Fiction vs. Physics Reality," physicists C. Efthimiou and S. Gandhi attempt to offer a mathematical proof that vampires and other creatures can't possibly exist. But as we shall see, their argument has a hole in it, large enough that you can drive a stake through it!

Here, we will deal mainly with the problem of the existence of vampires being creatures as presented in movies such as *Vampires* by John Carpenter or in the popular *Blade* trilogy. Efthimiou and Gandhi claim that the existence of such vampires would lead to the inevitable extinction of humans in next to no time. According to myth, vampires feed on human blood and after sucking from a non-vampire victim, the victim itself becomes a member of the vampire race for the rest of its life (or death, or living death, since it is known that the vampires are the members of a larger class of night creatures called *living dead*). To sum up, one meal for one vampire means that one human is gone and there is one more vampire in the world. This way, in a discrete time (e.g., having monthly updates on demographics), the simultaneous life (or death or…) of vampires and non-vampires could be modeled by a set of difference equations:

$$h(t+1) = h(t) - c\,v(t)$$
$$v(t+1) = v(t) + c\,v(t).$$

Here, $v(t)$ and $h(t)$ are the numbers of vampires and humans in the month t respectively, while c is the average monthly bloodsucking frequency (i.e., the average number of non-vampires one vampire consumes in one month).

For the set of equations to be properly defined, starting conditions $h(0)$ and $v(0)$ should be known. Efthimiou and Gandhi used quite advantageous assumptions for humanity: that in January 1600 (the authors present this date as the rough date of the first serious appearance of these dark creatures in folklore) there existed only one vampire against 500 million non-vampires, which was the world population at the time. Another assumption is that bloodsucking occurs only once per month, which is a very mild and cautious assumption since in the movies, you can see that vampires eat all the time. According to the given model, this would mean that $c = 1$, and therefore $v(t+1) = 2v(t)$. Thus, since $v(0) = 1$, we get a simple equation $v(t) = 2^t$, so the vampire population grows exponentially!

Consequently, since $v(29) = 2^{29}$ is greater than 500 million, we conclude that the human race would cease to exist some two and a half years after the appearance of the first vampire, approximately thirty years before Rene Descartes would get to think that he is since he thinks. Needless to say, vampires would also die out soon afterwards, because of the absence of human blood.

It seemed like Efthimiou and Gandhi gave undeniable proof and closed the ever exciting matter of vampire existence. However, the defiance of common folks was great. On the web site `digg.com`, one could read skeptical comments stating that the authors did not take into account that the extinction of the human race would not be allowed by Blade, Buffy and Chuck Norris; that garlic necklaces were quite popular in the seventeenth century; and that natural laws could not be applied to supernatural beings.

On second thought, the model has not accounted for the birth-rate of non-vampires and death-rate of vampires (actually the death-death-rate since they are already dead, but when they die again they should stay dead but stop being living) due to close encounters with stakes, garlic and holy water. Also, vampires are presented exclusively as greedy consumers: a rational strategy of managing their human resources is not considered.

In fact, such a rational policy was proposed over twenty years ago by a team of Austrian mathematicians, led by R. Hartl and A. Mehlmann. They used mathematical optimization and control theory to model a dynamic confrontation between the human race and vampires taking into account optimal bloodsucking strategies of the vampire population. The authors were irritated that this matter was only being considered by anthropologists and producers of low-budget movies. They presented a more realistic model of the conflict:

$$v' = -av + cv$$
$$h' = nh - cv.$$

Here, v' and h' are the rates of change of vampire and human population in time, a is the death-rate of vampires caused by stakes and other good stuff, while n is an increase in human population by common reproduction. As before, c is the bloodsucking frequency. This is an example of the more general Lotka-Volterra set of equations, common for modeling predator-prey interactions in biological systems. Human-vampire conflict is special, since the predator actually turns the prey into his own kind, thus increasing his own population. Needless to say, this was not encountered in any other biological system!

Hartl and Mehlmann put themselves in the position of vampires and tried to solve what was to be adequately named the Transylvanian problem: determine the optimal bloodsucking frequency $c(t)$ with respect to time, which would maximize the objective function representing total utility per vampire, assuming vampires have a utility function $U(c)$. The utility $U(c)$ is derived by the average vampire from blood consumption at rate c, while total utility per vampire is what determines joy and happiness inside the vampire community. This problem was solved for three different kinds of vampires:

(a) asymptotically satiated vampires, when $U'' < 0$;
(b) blood maximizing vampires, when $U'' = 0$;
(c) unsatiable vampires, when $U'' > 0$,

which, it should be noted, is also a large step forward in the chaotic classification of vampires. Clearly, Efthimiou and Ghandi have underestimated the mathematical capabilities of the vampire community. Strikingly, there are strategies for all three kinds of vampires which would approach a long-run equilibrium for the value of the stock of humans per vampire $x(t) = h(t)/v(t)$. In other words, vampires could have a plan to manage their human resources in order to keep both human race—and thus, their own—in existence. Later, these same scientists used Hopf bifurcation to strengthen their results and succeeded in modelling the human-vampire conflict so that optimal bloodsucking rates change periodically, which is more concurrent with archived empirical evidence in popular literature.

These works created swift reaction by the counter-vampire scientific community. Political economist, D. Snower expressed his indignation with the efforts of Hartl and others to help the vampires strengthen and consolidate their bloodsucking strategies, and came forth with a macroeconomic optimal strategy of the destruction of vampires. His philanthropic plan consists of a smart allocation of labor services producing economic goods onto the production of useful counter-vampire instruments, such as stakes, garlic, or rosary beads. However, by standard application of the maximum principle, Snower reached the surprising result that a complete destruction of vampires would not be socially optimal for humans either!

From the research done by Hartl, Mehlmann and Snower, it is clear that the relations between vampires and non-vampires are much more sophisticated than those presented by Efthimiou and Ghandi. Assuming that both vampires and non-vampires are intelligent beings, choosing their actions rationally, science actually sends a rather pacifist message that the optimal plan for both races is life (or death…) together. And that all should live (dead or not) happily ever after.

Except for the unfortunate fellows who get their blood sucked dry. ■

Further Reading

C. J. Efthimiou and S. Gandhi, Ghosts, Vampires and Zombies: Cinema Fiction vs Physics Reality, *arxiv:physics/060859*, 2006.

R. F. Hartl and A. Mehlmann, The Transylvannian Problem of Renewable Resources, *Revue Francaise d'Automatique, Informatique et de Recherche Operationelle*, Vol. 16 (1982), 379–390.

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