

This file is based on the answer key to a Math 310 Practice Exam.

1. Use truth tables to determine whether or not the following pairs of statements are logically equivalent.

(a) $[(p \wedge q) \rightarrow r]$ and $(p \rightarrow r) \wedge (q \rightarrow r)$

| p | q | r | $p \wedge q$ | $(p \wedge q) \rightarrow r$ |
|-----|-----|-----|--------------|------------------------------|
| T | T | T | T | T |
| T | T | F | T | F |
| T | F | T | F | T |
| T | F | F | F | T |
| F | T | T | F | T |
| F | T | F | F | T |
| F | F | T | F | T |
| F | F | F | F | T |

| p | q | r | $p \rightarrow r$ | $q \rightarrow r$ | $(p \rightarrow r) \wedge (q \rightarrow r)$ |
|-----|-----|-----|-------------------|-------------------|--|
| T | T | T | T | T | T |
| T | T | F | F | F | F |
| T | F | T | T | T | T |
| T | F | F | F | T | F |
| F | T | T | T | T | T |
| F | T | F | T | F | F |
| F | F | T | T | T | T |
| F | F | F | T | T | T |

Since the last columns of these truth tables are not identical, these two propositions are not logically equivalent.

(b) $p \wedge (q \vee r)$ and $(p \wedge q) \vee (p \wedge r)$

| p | q | r | $q \vee r$ | $p \wedge (q \vee r)$ |
|-----|-----|-----|------------|-----------------------|
| T | T | T | T | T |
| T | T | F | T | T |
| T | F | T | T | T |
| T | F | F | F | F |
| F | T | T | T | F |
| F | T | F | T | F |
| F | F | T | T | F |
| F | F | F | F | F |

| p | q | r | $p \wedge q$ | $p \wedge r$ | $(p \wedge q) \vee (p \wedge r)$ |
|-----|-----|-----|--------------|--------------|----------------------------------|
| T | T | T | T | T | T |
| T | T | F | T | F | T |
| T | F | T | F | T | T |
| T | F | F | F | F | F |
| F | T | T | F | F | F |
| F | T | F | F | F | F |
| F | F | T | F | F | F |
| F | F | F | F | F | F |

Since the last columns of these truth tables are identical, these two propositions are logically equivalent.