Math 487 - Foundations of Geometry
Day 10 Group Assignment **Name:**______________________________

Ruler Axioms and Betweenness:

Proposition 2.4: The Euclidean Plane satisfies the Ruler Postulate.

1. Prove that this theorem holds when ℓ is a vertical line.

Note: For the non-vertical case, we showed the the standard ruler is one-to-one and onto on DGW 9. TO complete the proof, it only remains to show that distance condition is satisfied. To see this, note that for points $P(x_1, y_1)$ and $Q(x_2, y_2)$ on a line ℓ given by $y = mx + b$, we have $d(P, Q) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
$= \sqrt{(x_2 - x_1)^2 + ((mx_2 + b) - (mx_1 + b))^2} = \sqrt{(x_2 - x_1)^2 + ((mx_2 - mx_1)^2} = \sqrt{(x_2 - x_1)^2 + m^2(x_2 - x_1)^2}$
$= |x_2 - x_1|\sqrt{1 + m^2} = |x_2\sqrt{1 + m^2} - x_1\sqrt{1 + m^2}|$, which is the difference between the standard ruler coordinates of P and Q.

Theorem 2.5 The Ruler Placement Postulate is not independent of the other axioms.

Take a few moments sometime between now and class on Monday to look at the outline of the proof of this Theorem in your textbook.

Notation: In order to abbreviate our notation, we will often write AB to represent the distance $d(A, B)$.

Definitions:

- A point B is **between points** A **and** C, denoted $A - B - C$, if $\{A, B, C\}$ is a collinear set of three distinct points and $AB + BC = AC$.
- A **line segment** is the union of two distinct points and all points between those two points, denoted either as segment AB or, preferably, as $\overline{AB}$. The points A and B are called the **endpoints** of segment $\overline{AB}$.
- Two segments are **congruent** if they have the same measure, denoted $\overline{AB} \cong \overline{CD}$.
- A point M is the **midpoint** of segment $\overline{AB}$ if $AM = MB$ and $\{A, M, B\}$ is collinear.
- A **bisector** of a segment is a line that contains the midpoint of the segment.
- A **ray** $\overrightarrow{AB}$ is the union of the segment $\overline{AB}$ and the set of all points C such that B is between A and C. The point A is called the **endpoint** of the ray $\overrightarrow{AB}$. (Note: ray $\overrightarrow{AB}$ and ray $\overrightarrow{BA}$ are different rays.)
- A **triangle** is the union of three segments determined by three noncollinear points, i,e., triangle ABC is the union of segment $\overline{AB}$, segment $\overline{AC}$, and segment $\overline{BC}$. Each of the three noncollinear points that determine a triangle is called a vertex of the triangle.

2. Let $P = (2, 1)$ and $Q = (5, 5)$ be points in the Cartesian Plane.

   (a) Find the midpoint of $\overline{PQ}$ in the Euclidean Plane.

(b) Find the midpoint of $\overline{PQ}$ in the Taxicab Plane.

(c) Find the midpoint of $\overline{PQ}$ in the Max-Distance Plane.

3. Look in your High School Geometry textbook and find the Axioms (if any) that correspond to Postulates 1, 2, 3, 4, 5a, and 6 and write them (or write "N/A") in the space provided below.

4. Explain why collinearity is necessary in the definition of betweenness (if possible, provide an example that illustrates your reasoning).