Math 487 - Foundations of Geometry
Day 11 Group Assignment **Name:**______________________________________

The Plane Separation Postulate:

Definition: A set S is **convex** if for every pair of distinct points P and Q in S, the segment $\overline{PQ}$ is a subset of S.

1. Draw an example of a set S in the Euclidean Plane that **is** convex and an example of a set T that is **not** convex.

Postulate 9(The Plane Separation Postulate) Given a line and a plane containing it, the points of the plane that do not lie on the line form two sets such that:

- each of the sets are convex
- if P is in one set and Q is in the other, then the segment $\overline{PQ}$ intersects the line.

Definition: Each of the two convex sets is called a **half-plane**, and the line is called the **edge**. Points in the same half-plane are said to be on the **same side**. Two points in different half-planes determined by a line are said to be on **opposite sides**.

Note: The line and the two half-planes associated with it are all disjoint sets.

2. Does the Cartesian Plane satisfy the Plane Separation Postulate? Justify your answer (you do not have to do a formal proof, but try to explain your reasoning as carefully as you can).

3. Give a specific counterexample that demonstrates that the Missing Strip Plane does not satisfy the Plane Separation Postulate.

Theorem 2.6(Pasch's Postulate) For any line ℓ and and triangle $\triangle ABC$, and any point D on ℓ such that $A-D-B$, either ℓ intersects $\overline{AC}$ or ℓ intersects $\overline{BC}$.

4. Draw examples in the Euclidean Plane that illustrate both options in Pasch's Postulate.

Claim: The Plane Separation Postulate and Pasch's Postulate are equivalent.

5. Explain what you would need to show in order to prove the claim given above. (You are not expected to complete a proof of this claim – just state how one would go about proving it).

6. Give a specific example in the Missing Strip Plane that shows that Pasch's Postulate does not hold.

Definitions: An **angle** is the union of two noncollinear rays with a common endpoint. The common endpoint is called the **vertex** of the angle, and the rays are called the **side**s of the angle.

Note: The Ruler Postulate and Ruler Placement Postulate were motivated by the "real-world" use of rulers. The SMSG Postulates 11-13 are motivated by the "real-world" use of a protractor.

Postulate 11:(Angle Measurement Postulate) To every angle there corresponds a real number between 0 and 180.

Postulate 12:(Angle Construction Postulate) Let $\overrightarrow{AB}$ be a ray on the edge of the half-plane H. For every r between 0 and 180, there is exactly one ray $\overrightarrow{AP}$ with P in H such that $m(\angle PAB) = r$.

Note: Here, $m(\angle PAB)$ denotes the measure of the angle $\angle PAB$ and $\angle PAB$ denotes the angle with sides $\overrightarrow{AB}$ and $\overrightarrow{AP}$ and with vertex A.

Postulate 13:(Angle Addition Postulate) If D is a point in the interior of $\angle BAC$, then $m(\angle BAC) = m(\angle BAD) + m(\angle DAC)$.

Note: Any angle has measure between 0 and 180. No angle has measure greater than or equal to 180, or less than or equal to 0.

7. Draw an example that illustrates Postulate 13.