Math 487 - Foundations of Geometry
Day 14 Group Assignment **Name:**______________________________

Postulate 15:(The SAS Postulate) Given a one-to-one correspondence between two triangles (or between a triangle and itself). If two sides and the included angle of the first triangle are congruent to the corresponding parts of the second triangle, then the correspondence is a congruence.

Definitions:

- An **isosceles triangle** is a triangle with two congruent sides. If the isosceles triangle has exactly two congruent sides, the angles opposite the two congruent sides are called **base angles**, the angle formed by the two congruent sides is called the **vertex angle**, and the third noncongruent side is called the **base**.
- An **equilateral triangle** is a triangle with all sides congruent. A **scalene triangle** has no congruent sides.

1. Draw and appropriately label a triangle of each of the different types defined above.

Theorem 2.10:(Pons Asinorum) The base angles of an isosceles triangle are congruent.

Proof: Let $\triangle ABC$ be an isosceles triangle with $\overline{AB} \cong \overline{AC}$. Since every angle has a unique angle bisector, let the ray $\overrightarrow{AD}$ be the bisector of $\angle BAC$. By the Crossbar Theorem, ray $\overrightarrow{AD}$ and segment $\overline{BC}$ intersect at a unique point E and $B - E - C$. Thus $\angle BAE = \angle BAD \cong \angle CAD = \angle CAE$. Since congruence of segments is an equivalence relation, $\overline{AE} \cong \overline{AE}$. Hence, by the SAS Postulate, $\triangle ABE \cong \triangle ACE$. Thus $\angle CBA = \angle EBA \cong \angle ECA = \angle BCA$. Therefore, the base angles of an isosceles triangle are congruent. $\square$.

2. Draw a diagram that illustrates the steps in the proof of Theorem 2.10.

3. Find specific examples that demonstrate that the SAS Postulate is not satisfied by the Taxicab Plane.

- Two lines are **parallel** if and only if they do not intersect.
- Given $\triangle ABC$, if $A - C - D$, then $\angle BCD$ is an **exterior angle** of $\triangle ABC$ Also, $\angle BAC$ and $\angle ABC$ are called **remote interior angles**.
- Given line $\overleftrightarrow{AB}$, line $\overleftrightarrow{DE}$, and line $\overleftrightarrow{BE}$ such that $A - B - C$, $D - E - F$, and $G - B - E - H$ where A and D on the same side of line $\overleftrightarrow{BE}$, then line $\overleftrightarrow{BE}$ is called a **transversal**. Angles $\angle ABE$ and $\angle FEB$ (also $\angle CBE$ and $\angle DEB$) are called **alternate interior angles**.

4. Draw a diagram that illustrates an exterior angle and the remote interior angles of a $\triangle ABC$.

5. Draw a pair of lines $\overleftrightarrow{AB}$ and line $\overleftrightarrow{DE}$, along with a transversal line $\overleftrightarrow{BE}$ satisfying the definition given above. You should also label a pair of alternate interior angles.

6. Draw a diagram illustrating the steps in the following proof of Theorem 2.11 (use the space to the right of the proof outline).

Theorem 2.11:(The Exterior Angle Theorem) Any exterior angle of a triangle is greater in measure than either of its remote interior angles.

Proof Outline: Let $\triangle ABC$ be given. Let D be a point such that $A - C - D$, and thus $\angle BCD$ is an exterior angle of $\triangle ABC$.

1. Let M be a midpoint of segment $\overline{BC}$.
2. $B - M - C$ and $\overline{BM} = \overline{MC}$
3. There is a point E on ray $\overrightarrow{AM}$ such that $A - M - E$ and $ME = MA$.
4. $\overline{ME} \cong \overline{MA}$.
5. $\angle AMB$ and $\angle EMC$ are vertical angles.
6. $\angle AMB \cong \angle EMC$
7. $\triangle AMB \cong \triangle EMC$
8. $\angle ABC = \angle ABM \cong \angle ECM = \angle ECB$.
9. $m(\angle ABC) = m(\angle ECB)$.
10. E and D are on the same side of $\overleftrightarrow{BC}$.
11. B, M, and E are on the same side of $\overleftrightarrow{CD}$.
12. $E \in \text{int}(\angle BCD)$.
13. $m(\angle BCE) + m(\angle ECD) = m(\angle BCD)$
14. $m(\angle BCD) > m(\angle BCE) = m(\angle ABC)$
15. The proof of the case for the other remote interior angle is similar. $\square$.