Math 487 - Foundations of Geometry
Day 16 Group Assignment **Name:**____________________________________

Recall:

- A **Saccheri quadrilateral** is a quadrilateral $ABCD$ where $\angle BAD$ and $\angle ABC$ are right angles and $\overline{AD} \cong \overline{BC}$. The segment $\overline{AB}$ is called the **base**, and the segment $\overline{CD}$ is called the **summit**.
- A **parallelogram** is a quadrilateral in which both pairs of opposite sides are parallel.
- A **rectangle** is a quadrilateral with four right angles.

Theorem 2.18: The segment joining the midpoints of the base and summit of a Saccheri quadrilateral is perpendicular to both the base and summit.

Theorem 2.19: The summit and base of a Saccheri quadrilateral are parallel.

Theorem 2.20: A Saccheri quadrilateral is a parallelogram.

Note: Although we will prove (See below) that a Saccheri quadrilaterals are parallelograms, they need not be rectangles. In fact, it can be shown that the existence of a rectangle is equivalent to the Euclidean Parallel Postulate (which does not hold in some models).

1. Prove Theorem 2.19.

2. Prove Theorem 2.20

3. Explain which part(s) of the definition of a rectangle may not hold in a Saccheri quadrilateral.

SMSG Postulate 16:(The Euclidean Parallel Postulate) Through a given external point there is at most one line parallel to a given line.

Playfair's Axiom: Through a point not on a line there is exactly one line parallel to the given line.

Theorem 2.21: In a neutral geometry, Euclid's Fifth Postulate is equivalent to the Euclidean Parallel Postulate.

4. In your own words, explain what you would need to show in order to prove of Theorem 2.21.

Note: Take a few moments between now and class on Monday to read the proof of Theorem 2.21 in your textbook.