The Elliptic Parallel Postulate: Any two lines intersect in at least one point.

Note: It is important to observe that elliptic geometry differs in an important way from Euclidean geometry and hyperbolic geometry. Euclidean geometry and hyperbolic geometry are neutral geometries with the addition of a parallel postulate. However, elliptic geometry cannot be a neutral geometry due to Theorem 2.14, which states that parallel lines exist in a neutral geometry. Hence, the Elliptic Parallel Postulate is inconsistent with the axioms of a neutral geometry. Elliptic geometry requires a different set of axioms in order for the axiomatic system to be consistent and contain an elliptic parallel postulate.

In a spherical model:

- Two lines (great circles) intersect in two points called poles or antipodal points.

- A line is unbounded, by this it is meant that a line has no endpoints. A great circle has no beginning and no end.

- A line has finite length.

- The concept of betweenness of points does not make sense. What does it mean for a point to be between two other points on a line (great circle)?

In order to formulate a consistent axiomatic system, several axioms from neutral geometry need to be dropped or modified.

- The incidence axiom that "any two points determine a unique line," needs to be modified to read "any two points determine at least one line."

- Betweenness of points may be replaced with additional axioms of separation that give properties of how points of a line separate each other.

- In spherical geometry, any two distinct lines intersect in more than one point, so the standard incidence axiom must be replaced with the "double elliptic geometry" axiom: two distinct lines intersect in two points.

Note: The Content and problems below are not intended to be done in class today. You should complete them and bring them to class on Friday.

1. Draw an illustration of a triangle in the Riemann Sphere that has three right angles.

2. Make a conjecture about the maximum and minimum angle sum for triangles in the Riemann Sphere.

3. Draw an illustration a Saccheri Quadrilateral on the Riemann Sphere. What, if anything can you say about the measure of the summit angles? What, if anything, can you say about the relationship between the lengths of the base and summit?