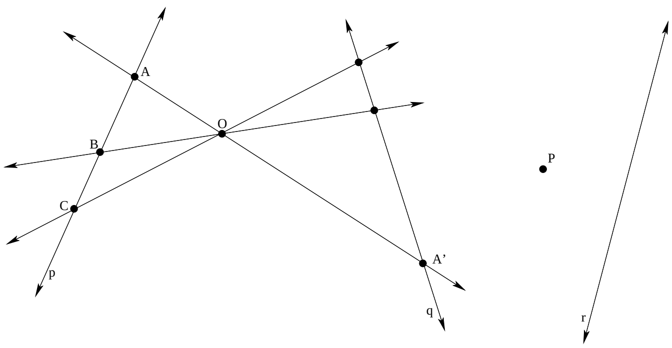
Math 487 - Foundations of Geometry
Day 29 Group Assignment **Name:**________________________________

**Plane Projective Geometry**

**Recall:**

- A one-to-one mapping between two pencils of points is called a **perspectivity** if the lines incident with the corresponding points of the two pencils are concurrent. The point where the lines intersect is called the **center of the perspectivity**. A perspectivity is denoted $X \overset{O}{\barwedge} X$ where O is the center of perspectivity.
- A one-to-one mapping between two pencils of points is called a **projectivity** if the mapping is a composition of finitely many perspectivities. A projectivity is denoted $X \wedge X'$.




1. In the diagram given above, label the points $B'$ and $C'$ on the line $q$ to complete the perspectivity $ABC \overset{O}{\barwedge} A'B'C'$. Then, add a pencil of points $A''$, $B''$, $C''$ to the line $r$ along with lines containing corresponding points to illustrate the perspectivity $A'B'C' \overset{P}{\barwedge} A''B''C''$. Note that combining these two perspectivities gives the projectivity $ABC \wedge A''B''C''$

   **Recall:** Axiomatic System for Plane Projective Geometry

   **Undefined Terms:** point, line, incident

   **Axiom 1:** Any two distinct points are incident with exactly one line.

   **Axiom 2:** Any two distinct lines are incident with at least one point.

   **Axiom 3:** There exist at least four points, no three of which are collinear.

   **Axiom 4:** The three diagonal points of a complete quadrangle are never collinear.

   **Axiom 5:** (Desargues' Theorem) If two triangles are perspective from a point, then they are perspective from a line.

   **Axiom 6:** If a projectivity on a pencil of points leaves three distinct points of the pencil invariant, it leaves every point of the pencil invariant.

   **Basic Theorems:**

   - **Theorem 4.1:** (Dual of Axiom 1) Any two distinct lines are incident with exactly one point.
   - **Theorem 4.2:** There exist a point and a line that are not incident.
   - **Theorem 4.3:** Every line is incident with at least three distinct points.
   - **Theorem 4.4:** Every line is incident with at least four distinct points.

2. Prove Theorem 4.1.

3. Prove Theorem 4.2. [Note: You may not assume the existence of any points or any lines.]

4. Prove the existence of a complete quadrangle.

5. Consider Axiom 1, Axiom 2, and Axiom 3. Show that these three axioms are independent.

6. Show Axiom 4 is independent of Axioms 1, 2, and 3. [Hint: A model from Chapter 1 might be useful.]

**Note:** The proofs of Theorems 4.3 and 4.4 are presentation eligible problems.