Math 487 - Foundations of Geometry
Day 38-39 Group Assignment **Name:**______________________________

Note: This assignment is designed to help you begin reviewing for our Final Exam. It will **not** be collected and graded.

Work on proving the following problems in groups. Once you feel you have a good argument, any member of your group can volunteer to present a problem to the rest of the class.

1. Consider the following axiomatic system:

   $A1$: For any two distinct points, there is exactly one line incident with both points.

   $A2$: For any two distinct lines, there is at most one point incident with both lines.

   $A3$: Every line is incident with at least two points.

   $A4$: There are 5 points in this geometry.

   $A5$: No point is incident with every line.

   (a) Find a model for this geometry. Be sure to explain how you know that each axiom holds.
   (b) Find a second model for this geometry that is not isomorphic to your previous model. Be sure to explain how you know that each axiom holds and how you know this it is not isomorphic to your previous model.
   (c) Which of the axioms in this model are independent? Justify your answer.
   (d) Write the duals to $A3$ and $A5$. Are these dual statements true statements in this axiomatic system?
   (e) Does this axiomatic system satisfy the principle of duality? Why or why not?

2. Determine whether or not the Missing Strip Plane satisfies the Euclidean Parallel Postulate.
3. Prove that a line segment has a unique midpoint.
4. Prove that triangle congruence is an equivalence relation.
5. Prove that the Missing Strip Place does not satisfy Pasch's Postulate.
6. Consider the following functions:

   (a) $f : \mathbb{R} \to \mathbb{R}$, $f(x) = x^2 + x$
   (b) $f : \mathbb{R} \to \mathbb{R}$, $f(x) = 2x$
   (c) $f : \mathbb{R} \to \mathbb{R}$, $f(x) = x - 3$
   (d) $g : \mathbb{R}^2 \to \mathbb{R}^2$, $g(x, y) = (x, y^2)$
   (e) $g : \mathbb{R}^2 \to \mathbb{R}^2$, $g(x, y) = (x^3, y)$
   (f) $g : \mathbb{R}^2 \to \mathbb{R}^2$, $g(x, y) = (5x, 2y)$
   (g) $g : \mathbb{R}^2 \to \mathbb{R}^2$, $g(x, y) = (-x, -y)$

   Which of these mappings are transformations? Which of these mappings are affine transformations?
   Which of these mappings are isometries? Justify your answers.

7. Given f, an isometry of $\mathbb{E}$ and a triangle $\triangle ABC$, if $f(A) = A'$, $f(B) = B'$, and $f(C) = C'$, then $\triangle ABC \cong \triangle A'B'C'$.
8. Find homogeneous coordinates for the line $\ell \begin{bmatrix} l_1 & l_2 & l_3 \end{bmatrix}$ containing the points $(-3, 1, 1)$ and $(1, -2, 1)$.
9. Find the point of intersection of the lines $\begin{bmatrix} 4 & -1 & 0 \end{bmatrix}$ and $\begin{bmatrix} 3 & 2 & -10 \end{bmatrix}$.
10. Find the angle between the lines $\begin{bmatrix} 4 & -1 & 0 \end{bmatrix}$ and $\begin{bmatrix} 3 & 2 & -10 \end{bmatrix}$.
11. Find the matrix for the transformation $R_{C,\theta}$ given that $C(-1, 2, 1)$ and $\theta = 30°$
12. Find the matrix for R_ℓ given that $\ell \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}$
13. Find the matrix for R_ℓ given that $\ell \begin{bmatrix} 1 & 1 & 1 \end{bmatrix}$