

Recall: The Axiomatic System for Fano's Finite Geometry.

Undefined Terms: point, line, incident

Axioms:

- **Axiom 1:** There exists at least one line
- **Axiom 2:** Every line has exactly three points incident to it.
- **Axiom 3:** Not all points are incident to the same line.
- **Axiom 4:** There is exactly one line incident with any two distinct points.
- **Axiom 5:** There is at least one point incident with any two distinct lines.

Claim: Fano's Geometry has exactly 7 points.

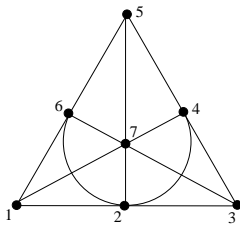
Fano's Theorem 3: Each point in Fano's geometry is incident with exactly three lines.

Note: You should take time to read and understand the proof of Theorem 3 (see the solution to Exercise 1.8 in your textbook).

Fano's Theorem 4: Fano's geometry consists of exactly seven lines.

Two Isomorphic Models for Fano's Geometry

- A diagrammatic model



- A tabular representation

Points	Lines
A, B, C, D, E, F, G	$ADB, AGE, AFC, BEC, BGF, CGD, FDE$

1. Verify that the models given above satisfy the axioms for Fano's Geometry.

2. Demonstrate that these models are isomorphic by constructing a mapping from one to the other.

3. Does this Geometry satisfy the principle of duality? How can you tell?

Axioms for a Finite Projective Plane:

Undefined Terms: point, line, incident

Defined Term: Points incident to the same line are *collinear*.

Axioms:

- **Axiom P1:** For any two distinct points, there is exactly one line incident with both points.
- **Axiom P2:** For any two distinct lines, there is at least one point incident with both lines.
- **Axiom P3:** Every line has at least three points incident with it.
- **Axiom P4:** There exist at least four distinct points of which no three are collinear.

Definition: A **projective plane of order n** is a geometry that satisfies the above axioms for a finite projective plane and has at least one line with exactly $n + 1$ ($n > 1$) distinct points incident with it.

Theorem P1: There exists a projective plane of order n for some positive integer n .

Claim: Fano's Geometry is a finite projective plane of order 2.

4. Complete the proof of Theorem P1 by verifying that one of the given models for Fano's Geometry is a model for Axioms P1 – P4.

Definition: Lines incident to the same point are *concurrent*.

5. Write the dual for each axiom of a finite projective plane.

6. Prove the Dual of Axiom P3