Math 487 - Foundations of Geometry
Day 8 Group Assignment **Name:**____________________________

More On Analytic Models and Incidence Axioms:

Recall:

In the Poincare Plane, lines are of two different types:

- vertical rays: ${}_a\ell = \{(x, y) \in H \mid x = a\}$.
- Type II lines ("semi-circles" with center point $(c, 0)$): ${}_c\ell_r = \{(x, y) \in H \mid (x - c)^2 + y^2 = r^2\}$

The distance between two points $P = (x_1, y_1)$, $Q = (x_2, y_2)$ is defined as follows: $d_H(P, Q) = \begin{cases} \left|\ln\left(\frac{y_2}{y_1}\right)\right| & \text{if } x_1 = x_2 \\ \left|\ln\left(\frac{\frac{x_2 - c + r}{y_2}}{\frac{x_1 - c + r}{y_1}}\right)\right| & \text{if } P, Q \text{ are on } {}_c\ell_r \end{cases}$

1. Consider the points $P = (2, 1)$ and $Q = (3, 5)$.

   (a) Find an equation for the line ${}_c\ell_r$ containing P and Q.

   (b) Find the $d_H(P, Q)$.

Recall:

Postulate 1 (Line Uniqueness) Given any two distinct points, there is exactly one line that contains them.

Postulate 5a (Existence of Points) Every plane contains at least three non-collinear points.

Theorem 2.2 Two distinct lines intersect in at most one point.

Note: There is one more axiom that, together with the previous two, make the Incidence Axioms.

Postulate 6 (Points on a Line Lie in a Plane) If two points lie in a plane, then the line containing these points lies in the same plane.

With these axioms in hand, our next question is to consider whether or not our analytic models satisfy these axioms. As we saw last time, several models do not satisfy line uniqueness.

Proposition 2.3 The Cartesian plane satisfies SMSG Postulates 1, 5(a), and 6.

2. Prove that the Cartesian Plane satisfies SMSG Postulate 1 (Hint: first consider the case when two points lie on a vertical line and then the case when they have different x-coordinates).

3. Noting that we proved that the Cartesian plane satisfies SMSG Postulate 5(a) in DGW 7 problem 4, explain why the Cartesian plane satisfies SMSG Postulate 6.

Corollary: The Euclidean plane, Taxicab plane, and Max-distance plane satisfy SMSG Postulates 1, 5(a), and 6.

4. Give a brief argument explaining why the Corollary stated above is true.

You may spend any remaining time discussing and working on the assigned homework with your group.