Math 487

Exam 2 Review Sheet

Postulates: Memorize the names of and be familiar with the meaning/content for each of the following SMSG postulates.

- Postulate 1: (Line Uniqueness) Given any two distinct points there is exactly one line that contains them.
- Postulate 2: (Distance Postulate) To every pair of distinct points there corresponds a unique positive number. This number is called the distance between the two points.
- Postulate 3: (Ruler Postulate) The points of a line can be placed in a correspondence with the real numbers such that:
  - To every point of the line there corresponds exactly one real number.
  - To every real number there corresponds exactly one point of the line.
  - The distance between two distinct points is the absolute value of the difference of the corresponding real numbers.
- Postulate 4: (Ruler Placement Postulate) Given two points P and Q of a line, the coordinate system can be chosen in such a way that the coordinate of P is zero and the coordinate of Q is positive.
- Postulate 5a: (Existence of Points) Every plane contains at least three non-collinear points.
- Postulate 6: (Points on a Line Lie in a Plane) If two points lie in a plane, then the line containing these points lies in the same plane.
- Postulate 9: (Plane Separation Postulate) Given a line and a plane containing it, the points of the plane that do not lie on the line form two sets such that:
  - each of the sets is convex
  - if P is in one set and Q is in the other, then segment $\overline{PQ}$ intersects the line.
- Postulate 11: (Angle Measurement Postulate) To every angle there corresponds a real number between 0 and 180.
- Postulate 12: (Angle Construction Postulate) Let $\overrightarrow{AB}$ be a ray on the edge of the half-plane H. For every r between 0 and 180, there is exactly one with P in H such that $m\,(\angle PAB) = r$.
- Postulate 13: (Angle Addition Postulate) If D is a point in the interior of $\angle BAC$, then $m\,(\angle BAC) = m\,(\angle BAD) + m\,(\angle DAC)$.
- Postulate 14: (Supplement Postulate) If two angles form a linear pair, then they are supplementary.
- Postulate 15: (SAS Postulate) Given a one-to-one correspondence between two triangles (or between a triangle and itself). If two sides and the included angle of the first triangle are congruent to the corresponding parts of the second triangle, then the correspondence is a congruence.
- Postulate 16: (Euclidean Parallel Postulate) Through a given external point there is at most one line parallel to a given line.

Notes:

- You should also be familiar with the Hyperbolic and Elliptic Parallel Postulates.
- You should also know which axioms are independent of the other axioms (and which are not).
- You should be familiar with which of the postulates each of our models do and do not satisfy and be able to provide justification and/or a counterexample to demonstrate whether or not a given postulate holds in a specific model.
- You should know the impact of the various parallel postulates on triangle angle sums, triangle congruence, rectangles, summit angles of Saccheri quadrilaterals, and triangle similarity.

Models: You should be familiar with each of the following components of geometric models.

- Understand how to compute distances in Discrete Planes, The Riemann and Modified Riemann Spheres, The Euclidean Plane, The Taxicab Plane, the Max-Distance Plane, the Missing Strip Plane, and the Poincare Half-Plane (I will provide formulas if I ask you to do computations related to the last two).
- Be able to determine whether or not a given function is a distance function.
- Understand how points and lines are defined in each of our models.
- Given two points, be able to find the distance between the points and the equation of the line containing the two points in each of the models.
- Given a standard ruler for one of our models, be able to compute coordinates, find midpoints, and modify the ruler to illustrate an application of the ruler placement postulate.

Definitions: You should be familiar with the definitions of the following terms (well enough to know and use them – no need to memorize verbatim).

- ruler, one to one, onto, line segment, ray, betweenness, midpoint
- equivalence relation, congruence of segments and angles
- the interior of an angle, right, acute, and obtuse angles, parallel and perpendicular lines
- angle bisector, perpendicular bisector, linear pair, supplementary angles, triangle congruence
- exterior angles, remote interior angles, transversal, alternate interior angles, Saccheri Quadrilateral, parallelogram, rectangle

Theorems: You should be familiar with the **statements** of the following Theorems.

- Theorem 2.6(Pasch's Postulate), Theorem 2.7, Theorem 2.9(The Crossbar Theorem), Theorem 2.11(Exterior Angle Theorem), Theorem 2.20

Theorems: You should be familiar with the **statements and proofs** of the following Theorems.

- Theorem 2.2, Theorem 2.8(The Vertical Angle Theorem), Theorem 2.10(Pons Asinorum), Theorem 2.13, Theorem 2.16, Theorem 2.17

Other Proofs:

For the exam I will also expect you to be able to:

- Prove 1 or 2 of the Euclidean Propositions from your text. You do not need to memorize them. I will provide the statements and will expect you to prove each theorem using postulates and other theorems from the course.
- Fill in the reasons for each step in the two column proof using postulates and theorems listed above.
- I may also ask you to prove a Theorem that is similar to one we looked at together, but new to you for the exam.