Math 311
Exam 1 Review Sheet

Propositional Logic and Quantifiers

• Understand the definition of a proposition and be able to determine whether or not a given statement is a proposition.
• Understand the definition and truth tables of the logical operators $\neg$, $\wedge$, $\vee$, $\Rightarrow$, and $\Leftrightarrow$
• Be able to build the truth table of a compound proposition involving 2 or 3 variables.
• Be able to translate back and forth between English statements and symbolic logical propositions.
• Understand the various ways of expressing a conditional statement in English (e.g. necessary, sufficient, only if, whenever, ...). Also know the difference between a conditional statement and its converse, inverse, and contrapositive statement.
• Understand what it means for two propositions to be logically equivalent and be able to prove the logical equivalence of a pair of propositions by building truth tables.
• Understand the what it means for a proposition to be a *tautology*, a *contradiction*, or a *contingency*.
• Be able to prove that a proposition is a tautology.
• Be able to negate propositions using De Morgan's Laws and other equivalences.
• Understand the definition of a predicate involving one or more variables.
• Understand the definition of the universal quantifier, the existential quantifier, and the uniqueness quantifier.
• Be able to determine the truth value of statements involving predicates and/or quantifiers.
• Understand the definition of a *counterexample* and how to use them to show that a statement is false.
• Understand how to find a "useful denial" of a statement involving predicates and quantifiers.
• Understand "hidden quantifiers" and how "mixed quantifiers" can work together to form a statement and the importance of the order of quantification in a statement.
• Be able to determine the truth value of statements involving multiple and "mixed" quantifiers.
• Be able to negate statements involving multiple and "mixed" quantifiers.

Basic Proof Techniques

• Understand and be able to apply logical argument forms such as *modus ponens*.
• Be able to read a mathematical statement and understand the "Given" and the "Goal" of the statement.
• Be able to write proofs by utilizing: Direct proof, Proof by Contraposition, and Proof by Contradiction.
• Memorize and be able to apply the definitions of odd numbers, even numbers, rational numbers, prime numbers, absolute value, and divisibility.
• Be able to use the Basic Properties of Integers and Real Numbers to prove related properties (you are **not** expected to memorize these).
• Understand and be able to show or make use of "closure properties" of integers, rational numbers, and real numbers.
• Understand and be able to carry out a "for all" proof.
• Understand and be able to carry out an "existence" proof.
• Understand and be able to carry out a "uniqueness" proof.
• Understand and be able to carry out proofs of "biconditional" statements.
• Understand and be able to carry out proofs of "or" statements.
• Understand and be able to utilize Proof by Cases (the Method of Exhaustion).
• Understand how to rule out unnecessary cases (without loss of generality statements).
• Understand the role of counterexamples in disproving statements.
• Understand how to use "backwards reasoning" to help find a proof for a given statement.
• Understand how to form conjectures and how to use examples to investigate a statement.