Math 127 - College Algebra
Handout: Algebraic Expressions

A. Definitions

• A **set** is a collection of objects of some type. We usually use capital letters to denote sets. Individual elements are called **elements**.

Examples: $\mathbb{N} = \{1, 2, 3, \ldots\}$, $\mathbb{W} = \{0, 1, 2, 3, \ldots\}$

Notation:

$a \in S$ [a is an element of the set S]
$b \notin S$ [b is *not* an element of the set S]
$S \subset T$ [S is a *subset* of the set T, that is, every element of S is in T]
$S = T$ [S and T have precisely the same elements]
$\emptyset$ [the empty set, a set that has *no* elements in it]

• A **constant** is a letter that is used to represent a *specific* element of a set. A **variable** is a letter that could refer to *any* element of a set.

• An **algebraic expression** is the result of applying mathematical operations [addition, subtraction, multiplication, division, exponents, etc.] to some collection of variables and real numbers.

• A **monomial** is an expression of the form ax^n, where n is a natural number and a is a real number.

• A **binomial** is an expression of the form $ax^n + bx^m$.

• A **trinomial** is an expression of the form $ax^n + bx^m + cx^l$

• A **polynomial** is any expression of the form $a_n x^n + a_{n-1}x^{n-1} + \ldots + a_1 x + a_0$.

Example: $3x^7 - 4x^5 + 12x^2 - 7x + 22$

• The **degree** of a polynomial is the exponent of the **leading term** [the monomial term with the highest exponent and a nonzero coefficient]. The **leading coefficient** is the constant of the leading term. Our example above has *degree* 7 and *leading coefficient* 3 since the *leading term* is $3x^7$.

Special Product and Factoring Formulas:

1. $(a - b)(a + b) = a^2 - b^2$ [Difference of Squares]
2. $(a \pm b)^2 = a^2 \pm 2ab + b^2$ [Perfect Square]
3. $(a \pm b)^3 = a^3 \pm 3a^2b + 3ab^2 \pm b^3$ [Perfect Cube]
4. $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$ [Sum of Cubes]
5. $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$ [Difference of Cubes]
6. $a^2 + b^2$ does not factor over the real numbers [Sum of Squares]