Math 127
Lines Handout
06/12/2008

Definition: A *linear equation* in two variables (x and y) is any equation that can be written in the form $ax + by = c$ with at least one of a and b non-zero. This form is called the *General Form* of the equation.

Not surprisingly, the graph of a linear equation is a straight line. Recall that the *graph* of an equation is the set of all points that *satisfy* the equation. We saw last time that we can graph linear equations by plotting a few points on the line. We find points on the line by substituting in a value for one of the variables and then and solving for the other. One particularly nice way to graph a line is to graph the *intercepts* of the line.

Slope: The slope of a line measures how "steep" the line is. It measures how much the line increases or decreases in the y direction when we move one unit the right in the x direction. We can find the slope m of a line by taking any two points on the line $P(x_1, y_1)$ and $Q(x_2, y_2)$ and using the formula:

$m = \dfrac{y_2 - y_1}{x_2 - x_1}$ or $m = \dfrac{rise}{run}$.

There are 3 main forms for the equation of a line:

1. General Form: $ax + by = c$
2. Slope/Intercept Form: $y = mx + b$ where m is the slope and b is the y coordinate of the y intercept.
3. Point/Slope Form: $y - y_1 = m(x - x_1)$ where m is the slope and $P(x, y)$ is a point on the line.

If we know two points on a line, we can find the slope of the line using the slope formula. If we know a point on the line and the slope, we can use either the slope/intercept form or the point/slope form equations to find an equation for the line.

Also, we can use algebra to put any linear equation into slope/intercept form so that we can just "read off" the slope and y-intercept of the line.

Horizontal and Vertical Lines:

Two special types of lines that do not quite fit the pattern above are horizontal and vertical lines. Horizontal lines run from left to right, and vertical lines go up and down.

Horizontal lines have equations of the form $y = c$ and so have slope zero (which makes sense since they do not "rise")

Vertical lines have equations of the form $x = c$ and so have "infinite" slope (which makes sense since they do not "run")

Parallel and Perpendicular Lines:

Two distinct lines are *parallel* to each other if they never intersect. For this to happen, the lines must have the same slope.

Two lines are *perpendicular* to each other if they form a right angle at the point where they intersect . For this to happen, the slopes of the lines must satisfy the equation:

$m_2 = -\frac{1}{m_1}$, or, written another way, $m_1 m_2 = -1$