Math 229
Algebra Review
05/30/2008

A. Exponents

Definition: $a^n = a \cdot a \cdot a \cdot a \cdot \ldots \cdot a$
(a multiplied by itself n times)

Properties:

1. $a^0 = 1$
2. $a^{-n} = \frac{1}{a^n}$
3. $a^m \cdot a^n = a^{m+n}$
4. $(a^m)^n = a^{mn}$
5. $(ab)^n = a^n b^n$
6. $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$
7. $\frac{a^m}{a^n} = a^{m-n} = \frac{1}{a^{n-m}}$
8. $\frac{a^{-m}}{b^{-n}} = \frac{b^n}{a^m}$
9. $\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n$

C. Algebraic Expressions
An **algebraic expression** is the result of applying mathematical operations to some collection of variables and real numbers.
• A **monomial** is an expression of the form ax^n, where n is a natural number and a is a real number.
• A **polynomial** is any expression of the form $a_n x^n + a_{n-1}x^{n-1} + \ldots + a_1 x + a_0$. When applying addition, subtraction, multiplication, and division to a polynomial, we simplify by combining "like terms".
Example: $3x^7 - 4x^5 + 12x^2 - 7x + 22$
Special Product and Factoring Formulas:

1. $(x-y)(x+y) = x^2 - y^2$
   [Difference of Squares]
2. $(x \pm y)^2 = x^2 \pm 2xy + y^2$
   [Perfect Square]
3. $(x \pm y)^3 = x^3 \pm 3x^2y + 3xy^2 \pm y^3$
   [Perfect Cube]
4. $x^3 + y^3 = (x+y)(x^2 - xy + y^2)$
   [Sum of Cubes]
5. $x^3 - y^3 = (x-y)(x^2 + xy + y^2)$
   [Difference of Cubes]
6. $x^2 + y^2$
   [Sum of Squares - Does not factor!]

B. Radicals

Definition: Suppose n is a positive integer and a is a real number. Then we define the ***n*th root of *a***, denoted by $\sqrt[n]{a}$ as follows:
• If $a = 0$, then $\sqrt[n]{a} = 0$.
• If $a > 0$ then $\sqrt[n]{a}$ is the *positive* real number b such that $b^n = a$.
• If $a < 0$ and n is **odd**, then $\sqrt[n]{a}$ is the *negative* real number b such that $b^n = a$.
• If $a < 0$ and n is **even**, then $\sqrt[n]{a}$ is not a real number, since there is no real number b such that $b^n = a$.
Examples:
(a) $\sqrt[2]{9} = \sqrt{9} = 3$ since $3 \cdot 3 = 9$.
(b) $\sqrt[3]{-8} = -2$ since $(-2)\cdot(-2)\cdot(-2) = -8$.
(c) $\sqrt{-16}$ is not a real number. (notice that $4 \cdot 4 = 16$, and $(-4)\cdot(-4) = 16$)
Properties:

1. $(\sqrt[n]{a})^n = a$ if $\sqrt[n]{a}$ is a real number.
2. $\sqrt[n]{a^n} = a$ if $a \geq 0$.
3. $\sqrt[n]{a^n} = a$ if $a < 0$ and n is odd.
4. $\sqrt[n]{a^n} = |a|$ if $a < 0$ and n is even.
5. $\sqrt[n]{ab} = \sqrt[n]{a}\sqrt[n]{b}$ provided both exist.
6. $\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$ provided both exist.
7. $\sqrt[m]{\sqrt[n]{a}} = \sqrt[mn]{a}$ provided both exist.

Warning!!
(a) In general, $\sqrt{a^2+b^2} \neq a + b$
(b) Also, in general, $\sqrt{a+b} \neq \sqrt{a} + \sqrt{b}$

D. Algebraic Fractions
• An **algebraic fraction** is the quotient of two polynomials.
• To simplify algebraic fractions, we "factor and reduce".
• To change from division to multiplication, we multiply by the reciprocal.
• When adding or subtracting fractions, we need to find a *common denominator*

Examples:
(a) $\dfrac{x-3}{x^2-1} \div \dfrac{x^2-x-6}{x+1} = \dfrac{x-3}{x^2-1} \cdot \dfrac{x+1}{x^2-x-6}$
$= \dfrac{x-3}{(x+1)(x-1)} \cdot \dfrac{x+1}{(x-3)(x+2)} = \dfrac{1}{(x-1)(x+2)}$

(b) $\dfrac{2x}{x-2} - \dfrac{4}{+3} = \dfrac{2x(x+3)}{(x-2)(x+3)} - \dfrac{4(x-2)}{(x-2)(x+3)}$
$= \dfrac{2x^2+6x-4x+8}{(x-2)(x+3)} = \dfrac{2x^2+2x+8}{(x-2)(x+3)} = \dfrac{2(x^2+x+4)}{(x-2)(x+3)}$