Math 229
Section 11.4: Marginal Functions in Economics
06/10/2008

Cost, Revenue, and Profit Marginal Functions

A. Cost Functions

$C(x)$: How much does it cost to produce x units?
$C'(x)$ = Marginal Cost: Approximately how much would it cost to produce the $(x+1)$st unit?
$\overline{C}(x) = \frac{C(x)}{x}$: What is the average cost per unit, including overhead, when x total units are produced?
$\overline{C'}(x)$: How much is the average cost per unit changing when x total units are produced? That is, how much would the average cost change if one more unit were produced?
Example:
Suppose $C(x) = 3000 + 10x - .01x^2$ is the cost function for producing widgets in a factory.
Notice $C(100) = 3900$, so it costs $3900 to produce the first 100 widgets.
$C'(x) = 10 - .02x$, and $C'(100) = 10 - 2 = 8$, so the 101st widget costs approximately $8 to produce.
$\overline{C}(x) = \frac{C(x)}{x} = \frac{3000}{x} + 10 - .01x$, and $\overline{C'}(x) = -3000x^{-2} - .01$
Also, $\overline{C}(100) = 39$, so including overhead, the average cost of producing 100 widgets is $39 per widget.
Since $\overline{C'}(100) = -.31$, producing 101 widgets would decrease the average cost per widget by about 31 cents get widget.

B. Revenue and Profit Functions

Given a demand equation, $p = f(x)$, where x is the number of units sold, and p is the price at which the units are sold:
Revenue: $R(x) = \text{(price)}\cdot\text{(quantity)} = p \cdot q = f(x) \cdot x$. How much total revenue is brought in when x units are sold?
$R'(x)$ = Marginal Revenue - How much revenue would be added or lost if one additional unit were sold when x units have already been sold?
Profit: $P(x) = \text{(Revenue) - (Cost)} = R(x) - C(x)$. How much profit (or loss) is there when x units are sold?
$P'(x)$ = Marginal Profit: How much profit would be gained or lost by selling one additional unit when x units have already been sold?
Example:
Suppose for our widget factory, the demand equation for widget is currently given by: $p = 270 - .005x$.
Then $R(x) = p \cdot x = 270x - .005x^2$, and $R'(x) = 270 - .01x$.
Similarly, $P(x) = R(x) - C(x) = (270x - .005x^2) - (3000 + 10x - .01x^2) = 260x + .005x^2 - 3000$, and $P'(x) = 260 + .01x$.
Notice that $R(100) = 26,950$, so our revenue would be $26,950 when 100 widgets are sold.
Also, $R'(100) = 269$, so if one more widget were sold, our revenue would increase by about $269.
Similarly, $P(100) = 22,950$, and $P'(100) = 261$, so when 100 widgets are sold, our net profit is $22,950, and if one more widget were sold, our profits would increase by about $261.