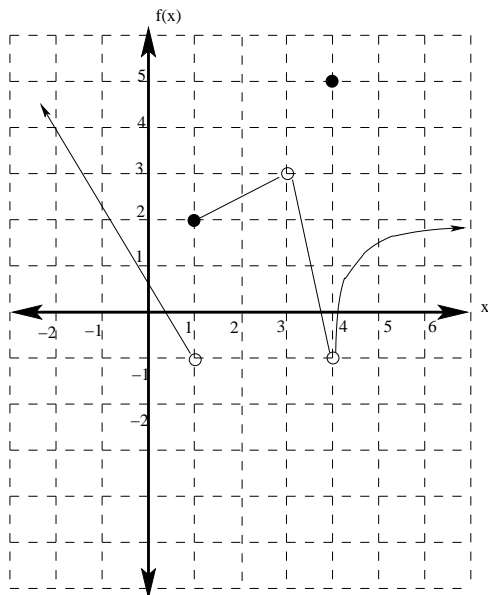Math 229
Exam 2 Practice Problems

1. Evaluate the following limits. Be sure to show enough work to justify your answers.

   (a) $\lim\limits_{x \to 0} \dfrac{x^2 - 2x}{2x^2 - x - 6}$

   (b) $\lim\limits_{x \to 2} \dfrac{x^2 - 2x}{2x^2 - x - 6}$

   (c) $\lim\limits_{x \to 2} \dfrac{x}{x - 2}$

   (d) $\lim\limits_{x \to \infty} \dfrac{x^2 - 2x}{2x^2 - x - 6}$

   (e) $\lim\limits_{x \to 0} \dfrac{2x^2 - x - 1}{x^2 - 1}$

   (f) $\lim\limits_{x \to 1} \dfrac{2x^2 - x - 1}{x^2 - 1}$

   (g) $\lim\limits_{x \to \infty} \dfrac{2x^2 - x - 1}{x^2 - 1}$

2. Given the following graph:

   (a) Find $\lim\limits_{x \to 1^-} f(x)$

   (b) Find $\lim\limits_{x \to 1^+} f(x)$

   (c) Find $\lim\limits_{x \to 4} f(x)$

   (d) Find $\lim\limits_{x \to \infty} f(x)$

   (e) List all points where $f(x)$ is discontinuous. Explain what goes wrong at each point.

3. Given the function

$$f(x) = \begin{cases} 3x - 2 & \text{if } x < 1 \\ 4 & \text{if } x = 1 \\ x^2 - 1 & \text{if } x > 1 \end{cases}$$

   (a) Graph $f(x)$.

   (b) Find $\lim\limits_{x \to 1} f(x)$.

   (c) Is $f(x)$ continuous at $x = 1$? Justify your answer.

4. Use the limit definition of the derivative to compute the derivative function $f'(x)$ if $f(x) = 5x^2 - 3x - 7$

5. Use the limit definition of the derivative to compute the derivative function $f'(x)$ if $f(x) = 4 - 2x - 3x^2$

6. Suppose $f(x) = x^3 - 3x^2 + 5$.

    (a) Find the equation for the tangent line to $f(x)$ when $x = 1$.

    (b) Find the value(s) of x for which the tangent line to $f(x)$ is horizontal.

7. Suppose $f(x) = (x+1)^{\frac{3}{2}}$.

    (a) Find the equation for the tangent line to $f(x)$ when $x = 3$.

    (b) Find the value(s) of x for which the tangent line to $f(x)$ is horizontal.

8. Find the derivative of each of the following functions. You **do not** have to use the limit definition, and you **do not** need to simplify your answers.

    (a) $h(x) = x^3 + \sqrt{x^3}$

    (b) $f(x) = 5x^4 - 3x^2 + \frac{2}{x}$

    (c) $h(x) = \dfrac{5x^3 - 4x^2 + 7x}{x^2}$

    (d) $h(x) = (x^2 - 4x^3)(4x^3 + 3x^2 - 7x + 3)$

    (e) $f(x) = (2x^2 + 5x - 4)(x^3 + 2x^2 - 1)$

    (f) $f(x) = \dfrac{2x + 3}{x^2 - 1}$

    (g) $h(x) = (x^3 - 2x + 1)^{\frac{5}{2}}$

    (h) $f(x) = \sqrt{2x^2 + 1}$

    (i) $\left(\dfrac{2 - 4x^3}{x^2 - 1} \right)^4$

    (j) $f(x) = (x^2 + 1)(x^3 - 2x + 1)^{\frac{3}{2}}$

9. Suppose you own a company that manufactures widgets, and the demand equation for them is given by $3x + 4p = 120$.

    (a) Find the revenue function $R(x)$, and use it to compute $R(10)$ and $R(40)$.

    (b) Find the marginal revenue function $R'(x)$

    (c) Compute $R'(10)$ and $R'(40)$ and explain what these numbers mean in practical terms.

    (d) If $C(x) = 20x + \frac{1}{4}x^2 + 100$, find $P(x)$ and use it to compute $P(10)$.

    (e) Find the marginal profit function $P'(x)$, use it to compute $P'(5)$, and explain what this means in practical terms.

10. Suppose you own a company that manufactures snow globes, and the demand equation for them is given by $5x + 4p = 200$.

    (a) Find the revenue function $R(x)$, and use it to compute $R(10)$ and $R(30)$.

    (b) Find the marginal revenue function $R'(x)$

    (c) Compute $R'(10)$ and $R'(30)$ and explain what these numbers mean in practical terms.