Math 229
Exam 3 Practice Problems

1. (a) Find the third derivative of $f(x)$, given that $f(x) = \dfrac{3}{x^2}$
   (b) Find the second derivative of $g(x)$, given that $g(x) = \dfrac{2x+1}{3x+2}$
   (c) Find the second derivative of $g(x)$, given that $h(x) = (1-x^2)^7$

2. Determine whether the following statements are True or False. Write a brief explanation to justify your answer.
   (a) If $f'(a) = 0$ and $f''(a) < 0$, then $(a, f(a))$ is a relative minimum of the function $f(x)$.
   (b) If $f'(x) > 0$ for $a \leq x \leq b$, then $(a, f(a))$ is an absolute minimum for $f(x)$ on $[a, b]$.
   (c) If $f(a)$ is undefined, then $x = a$ is a vertical asymptote of $f(x)$.
   (d) If $f(x) = \dfrac{p(x)}{q(x)}$, where both $p(x)$ and $q(x)$ are polynomials of the same degree, then $f(x)$ has a non-zero horizontal asymptote.
   (e) The absolute maximum of a function $f(x)$ on an interval $[a, b]$ must occur when $x = a$, when $x = b$, or at a critical point of f inside the interval $[a, b]$.

3. Let $f(x) = x^4 - 8x^3 + 16x^2$
   (a) Find the x and y intercepts of $f(x)$.
   (b) Find the intervals where $f(x)$ is increasing and the intervals where $f(x)$ is decreasing.
   (c) Find and classify the relative extrema of $f(x)$.
   (d) Find the equation of the tangent line to $f(x)$ when $x = 1$.

4. Given that $f(x) = \frac{1}{5}x^5 - \frac{2}{3}x^3$:
   (a) Find the intervals where $f(x)$ is concave up and the intervals where $f(x)$ is concave down.
   (b) Find the coordinates of the inflection points of $f(x)$.

5. Find the horizontal and vertical asymptotes (if any) of the following functions. (You do **not** need to sketch the graphs)
   (a) $f(x) = \dfrac{2x-3}{x^2-1}$.
   (b) $f(x) = \dfrac{3x^2-3x}{x^2-1}$.

6. Carefully draw the graph of a function satisfying the following conditions:
   x-intercepts: $(-2, 0), (2, 0), (5, 0)$; y-intercept: $(0, 5)$
   Increasing on $(-\infty, 0) \cup (3, \infty)$ and Decreasing on $(0, 3)$
   Concave Up on $(-1, 0) \cup (2, \infty)$ and Concave Down on $(-\infty, -1) \cup (0, 2)$
   $f(-1) = 3$, and $f(3) = -4$.

7. Let $f(x) = \frac{1}{4}x^4 - x^3$ (Use top of the next page if you need additional space for your graph)
   (a) Find the x and y intercepts of $f(x)$.
   (b) Find the intervals where $f(x)$ is increasing and those where $f(x)$ is decreasing.
   (c) Find and classify the relative extrema of $f(x)$.
   (d) Find the intervals where $f(x)$ is concave up and those where $f(x)$ is concave down.
   (e) Find any inflection points of $f(x)$.
   (f) Graph $f(x)$. Be sure to label all relative extrema, intercepts, and inflection points.

8. Suppose the daily cost for producing x widgets is given by $C(x) = 5x^2 - 20x + 500$, where $C(x)$ is in dollars, and a maximum of 20 widgets can be produced each day.

   (a) Find the production level which minimizes the daily costs. Also find the daily cost at this production level.

   (b) Find the production level which minimizes the **average** cost per widget. Also find the average cost per sidget at this production level.

9. Suppose the daily cost and revenue for producing x widgets are given by the functions: $C(x) = 750 - 3x + .005x^2$ and $R(x) = 825 + 2x - .005x^2$ for $0 \leq x \leq 400$. Find the production level which maximizes daily **profits**. Also find the amount of profit at this production level.

10. (16 points) Suppose you own a furniture store and sign a contract with a retailer to supply chairs. The terms of the contract state that you will charge $90 per chair when up to 300 chairs are ordered, but you will reduce the price by 25 cents per chair (on the entire order) for every chair ordered over 300, up to a total of 100 additional chairs. What is the largest revenue you can make under this contract? [Hint: Consider the cases $0 \leq x \leq 300$ and $300 < x \leq 400$ separately]