Math 229
Final Exam Practice Problems

1. Given that $f(x) = \sqrt{3-2x}$ and $g(x) = \dfrac{4}{3x-6}$

   (a) Find $f(-3)$ and $g(3a+2)$

   (b) Find $\dfrac{g}{f}(1)$

   (c) Find $f \circ g(1)$

   (d) Find the domain of $fg(x)$? Give your answer in interval notation.

2. The S. Claus Toy Company is working on a shipment of sleds that needs to ship at the end of the week. Since his workers are very inexpensive and his material costs are low, his overhead costs are only \$50 a day, and it only costs \$25 in parts and labor to make each sled. Based on a complete analysis of who has been naughty and who has been nice, Mr. Claus has determined that the daily demand equation for sleds is given by the equation $2p + 4x = 100$, where p is the price per sled in dollars, and x is the number of sleds sold each day.

   (a) Find an equation $C(x)$ the daily cost function for sled production, where x is the number of sleds made each day in the workshop.

   (b) Find the daily revenue function, $R(x)$, assuming that Mr. Claus is compensated for the sleds he makes based on the demand equation given above.

   (c) Find the daily profit function, $P(x)$.

   (d) Find the marginal profit function.

   (e) Find the maximum profit that S. Claus can make each day, assuming that his workshop can produce at most 10 sleds per day (they have to spend time making other types of toys, you know!)

3. Evaluate the following limits:

   (a) $\lim\limits_{x\to 0} \dfrac{x^2-x-2}{x^2-4}$

   (b) $\lim\limits_{x\to 2} \dfrac{x^2-x-2}{x^2-4}$

   (c) $\lim\limits_{x\to \infty} \dfrac{x^2-x-2}{x^2-4}$

4. Use the limit definition of the derivative to compute the derivative function $f'(x)$ if $f(x) = x^2 + 3x - 2$. You must show all work to receive credit for this problem.

5. Given the function:

$$f(x) = \begin{cases} x & \text{if } x < 2 \\ 3 & \text{if } x = 2 \\ 6 - x & \text{if } x > 2 \end{cases}$$

   (a) Graph $f(x)$.

   (b) Find $\lim\limits_{x\to 2^-} f(x)$ and $\lim\limits_{x\to 2^+} f(x)$.

   (c) Is $f(x)$ continuous at $x = 1$? Justify your answer.

6. Use properties of logarithms to expand the following expression:

$$\ln\left(\frac{x^3 z}{\sqrt[5]{xy}}\right)$$

7. Find the exact value of each of the following:

   (a) $\log_3(\frac{1}{27})$

   (b) $\log_9(\frac{1}{27})$

   (c) $\log_{17}(1)$

   (d) $5^{\log_5(472)}$

8. Find the interest rate needed for an investment of \$5,000 to triple in 10 years if the interest is compounded continuously.

9. Find the derivative of each of the following functions. You **do not** have to use the limit definition, and you **do not** need to simplify your answers.

   (a) $f(x) = 5x^4 - 3\sqrt{x} + \ln(2x - 3)$

   (b) $f(x) = (x^2 + 2)e^{2-3x}$

   (c) $f(x) = \dfrac{2x - 5}{x^2 + 1}$

   (d) $f(x) = (4x^4 - 7x + 5)^{\frac{3}{2}}$

10. Let $f(x) = \frac{3}{4}x^4 + x^3 - 9x^2 + 12$

    (a) Find the intervals where $f(x)$ is increasing and those where $f(x)$ is decreasing.

    (b) Find and classify the relative extrema of $f(x)$.

    (c) Find the equation for the tangent line to $f(x)$ when $x = 2$.

11. Sketch the graph of a function that satisfies the following conditions:

    Domain: $(-\infty, \infty)$, x-intercepts: $(-4, 0), (2, 0), (8, 0)$, y-intercept: $(0, -3)$

    Increasing on: $(-2, 5)$, Decreasing on: $(-\infty, -2) \cup (5, \infty)$

    Concave up on: $(-\infty, 0) \cup (2, 4)$, Concave down on: $(0, 2) \cup (4, \infty)$

    Local Max: $(5, 5)$, Local Min: $(-2, -5)$, Inflection Points: $(0, -3), (2, 0), (4, 3)$

12. If the marginal cost of a manufactoring process is known to be $45x^2 + 32x - 700$, and the fixed costs are \$2000, find the cost function $C(x)$.

13. Evaluate the following integrals:

    (a) $\displaystyle\int 6\sqrt{x} - 4x^{-1}\, dx$

    (b) $\displaystyle\int_0^4 2e^{2x} + x^{\frac{1}{2}}\, dx$

14. Find the average value of $f(x) = x^3 - \frac{1}{x^2}$ for $-2 \leq x \leq 2$.

15. Given that:

$$A = \begin{bmatrix} 1 & -5 \\ 3 & 0 \end{bmatrix} B = \begin{bmatrix} 2 & -3 \\ 1 & 7 \end{bmatrix} C = \begin{bmatrix} 1 & 3 & -2 \\ 2 & 0 & 4 \end{bmatrix} D = \begin{bmatrix} 0 & \frac{1}{3} \\ -\frac{1}{5} & \frac{1}{15} \end{bmatrix}$$

    (a) Find $2A - B$

    (b) Find BC

    (c) Prove that $DA = AD$.

16. Use matrices to solve:

$$\begin{cases} x + 3y + z = 3 \\ 3x + 8y + 3z = 7 \\ 2x - 3y + z = -10 \end{cases}$$

17. The owner of a luxury yacht that sails among the Greek islands charges \$600 per person per day if exactly 20 people sign up for the cruise. However, if more than 20 people sign up for the cruise (up to the maximum capacity of 90 people), then the fare for **every** passenger is reduced by \$4 per day for each passenger in excess of 20. The cruise will not run if fewer than 20 passengers sign up. Find the number of passengers that maximizes the daily revenue for the cruise. Also find the maximum revenue, and the price per person when revenue is maximized.