

A. Limits: The function f has **limit** L as x approaches a , written $\lim_{x \rightarrow a} f(x) = L$ if the value of $f(x)$ can be made as close to the number L as we please by taking x sufficiently close to (but not equal to) a .

Example: $\lim_{x \rightarrow 1} 5x - 2 = 3$

B. Properties of Limits: Suppose $\lim_{x \rightarrow a} f(x) = L$ and $\lim_{x \rightarrow a} g(x) = M$. Then:

1. $\lim_{x \rightarrow a} [f(x)]^r = [\lim_{x \rightarrow a} f(x)]^r = L^r$
2. $\lim_{x \rightarrow a} c \cdot f(x) = c \cdot \lim_{x \rightarrow a} f(x) = c \cdot L$
3. $\lim_{x \rightarrow a} [f(x) \pm g(x)] = \lim_{x \rightarrow a} f(x) \pm \lim_{x \rightarrow a} g(x) = L \pm M$
4. $\lim_{x \rightarrow a} [f(x)g(x)] = [\lim_{x \rightarrow a} f(x)] \cdot [\lim_{x \rightarrow a} g(x)] = L \cdot M$
5. $\lim_{x \rightarrow a} \frac{f(x)}{g(x)} = \frac{\lim_{x \rightarrow a} f(x)}{\lim_{x \rightarrow a} g(x)} = \frac{L}{M}$

C. Limits at Infinity: The function f has **limit** L as x increases without bound (or as x approaches infinity), written $\lim_{x \rightarrow \infty} f(x) = L$ if the value of $f(x)$ can be made as close to the number L as we please by taking x to be sufficiently large. Similarly, the function f has **limit** L as x decreases without bound (or as x approaches negative infinity), written $\lim_{x \rightarrow -\infty} f(x) = M$ if the value of $f(x)$ can be made as close to the number M as we please by taking x to be sufficiently large in the negative direction.

Example: $\lim_{x \rightarrow \infty} \frac{3x + 1}{x - 10} = 3$

D. One Sided Limits: The function f has **right-hand limit** L as x approaches a from the right, written $\lim_{x \rightarrow a^+} f(x) = L$ if the value of $f(x)$ can be made as close to the number L as we please by taking x to be sufficiently close to a but greater than a . Similarly, the function f has **left-hand limit** L as x approaches a from the left, written $\lim_{x \rightarrow a^-} f(x) = L$ if the value of $f(x)$ can be made as close to the number L as we please by taking x to be sufficiently close to a but less than a .

E. Continuity: A function f is **continuous at the point** $x = a$ if the following hold:

1. $f(a)$ is defined
2. $\lim_{x \rightarrow a} f(x)$ exists (that is both its one-sided limits exist and agree)
3. $\lim_{x \rightarrow a} f(x) = f(a)$

Notes:

1. Constant functions and polynomial functions are continuous everywhere.
2. Rational functions $R(x) = \frac{f(x)}{g(x)}$, with f and g polynomials are continuous except where $g(x) = 0$.
3. If two functions f and g are continuous at a point a , then so are: $f \pm g$, fg , and $\frac{f}{g}$ (provided $g(a) \neq 0$).