Math 127
Systems of Equations and Matrix Practice Problem Solutions

1. Solve the following systems of equations using substitution:

   (a) $\begin{cases} 7x + 4y = 29 \\ 2x - y = 4 \end{cases}$
   First, we solve the second equation for y. This gives $y = 2x - 4$.
   Next, we substitute this into the first equation, yielding: $7x + 4(2x - 4) = 29$
   Solving this for x: $7x + 8x - 16 = 29$, or $15x = 45$. Thus $x = 3$.
   Finally, $y = 2(3) - 4 = 6 - 4$, so $y = 2$. Thus our solution is $(3, 2)$.
   Check: $7(3) + 4(2) = 21 + 8 = 29$ and $2(3) - 2 = 6 - 2 = 4$

   (b) $\begin{cases} x^2 + y = 100 - 2x \\ 2y - 16x = 50 \end{cases}$
   First, we solve the second equation for y. This gives $2y = 16x + 50$, or $y = 8x + 25$.
   Next, we substitute this into the first equation, yielding: $x^2 + (8x + 25) = 100 - 2x$.
   Simplifying, we have: $x^2 + 8x + 25 + 2x - 100 = 0$, or $x^2 + 10x - 75 = 0$
   This factors to give: $(x + 15)(x - 5) = 0$, so either $x = -15$ or $x = 5$
   If $x = -15$, then $y = 25 + 8(-15) = -95$. If $x = 5$, then $y = 25 + 8(5) = 65$
   Therefore our solutions are: $(-15, -95)$ and $(5, 65)$
   Check: $(-15)^2 + (-95) = 225 - 95 = 130$ while $100 - 2(-15) = 130$
   Check: $(5)^2 + (65) = 25 + 65 = 90$ while $100 - 2(5) = 90$

2. Solve the following systems of equations using elimination:

   (a) $\begin{cases} 7x - 8y = 9 \\ 4x + 3y = -10 \end{cases}$
   We first multiply the first equation by 4 and the second by -7 yielding:
   $\begin{cases} 4[7x - 8y] = 4[9] \\ -7[4x + 3y] = -7[-10] \end{cases}$ or $\begin{cases} 28x - 32y = 36 \\ -28x - 21y = 70 \end{cases}$
   Adding these gives: $-53y = 106$, or $y = -2$. Substituting back into the original first equation: $7x + 16 = 9$, or $7x = -7$, so $x = -1$.
   Therefore, our solution is $(-1, -2)$. Check: $7(-1) - 8(-2) = -7 + 16 = 9$ and $4(-1) + 3(-2) = -4 - 6 = -10$.

   (b) $\begin{cases} 3x - 2y = 7 \\ 5x + 7y = -5 \end{cases}$
   We first multiply the first equation by 7 and the second by 2 yielding:
   $\begin{cases} 7[3x - 2y] = 7[7] \\ 2[5x + 7y] = 2[-5] \end{cases}$ or $\begin{cases} 21x - 14y = 49 \\ 10x + 14y = -10 \end{cases}$
   Adding these gives: $31x = 39$, or $x = \frac{39}{31}$. Substituting back into the original first equation: $3(\frac{39}{31}) - 2y = 7$, or $\frac{117}{31} - 7 = \frac{117}{31} - \frac{217}{31} = -\frac{100}{31} = 2y$, so $y = -\frac{50}{31}$.
   Therefore, our solution is $(\frac{39}{31}, -\frac{50}{31})$. Check: $3(\frac{39}{31}) - 2(-\frac{50}{31}) = \frac{117}{31} + \frac{100}{31} = \frac{217}{31} = 7$ and $5(\frac{39}{31}) + 7(-\frac{50}{31}) = \frac{195}{31} - \frac{350}{31} = -\frac{155}{31} = -5$

3. Given that:
   $$A = \begin{bmatrix} 1 & -5 \\ 3 & 0 \end{bmatrix} B = \begin{bmatrix} 2 & -3 \\ 1 & 7 \end{bmatrix} C = \begin{bmatrix} 1 & 3 & -2 \\ 2 & 0 & 4 \end{bmatrix} D = \begin{bmatrix} 0 & \frac{1}{3} \\ -\frac{1}{5} & \frac{1}{15} \end{bmatrix}$$

   (a) Find $2A - B$
   $= \begin{bmatrix} 2 & -10 \\ 6 & 0 \end{bmatrix} - \begin{bmatrix} 2 & -3 \\ 1 & 7 \end{bmatrix} = \begin{bmatrix} 0 & -7 \\ 5 & -7 \end{bmatrix}$

   (b) Find BC
   $= \begin{bmatrix} 2 & -3 \\ 1 & 7 \end{bmatrix} \cdot \begin{bmatrix} 1 & 3 & -2 \\ 2 & 0 & 4 \end{bmatrix} = \begin{bmatrix} (2-6) & (6+0) & (-4-12) \\ (1+14) & (3+0) & (-2+28) \end{bmatrix} = \begin{bmatrix} -4 & 6 & -16 \\ 15 & 3 & 26 \end{bmatrix}$

(c) Prove that $DA = AD$.

$$DA = \begin{bmatrix} 0 & \frac{1}{3} \\ -\frac{1}{5} & \frac{1}{15} \end{bmatrix} \cdot \begin{bmatrix} 1 & -5 \\ 3 & 0 \end{bmatrix} = \begin{bmatrix} (0+1) & (0+0) \\ (-\frac{1}{5}+\frac{1}{5}) & (1+0) \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$AD = \begin{bmatrix} 1 & -5 \\ 3 & 0 \end{bmatrix} \cdot \begin{bmatrix} 0 & \frac{1}{3} \\ -\frac{1}{5} & \frac{1}{15} \end{bmatrix} = \begin{bmatrix} (0+1) & (\frac{1}{3}-\frac{1}{3}) \\ (0+0) & (1+0) \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

4. Let $A = \begin{bmatrix} 2 & 2 \\ -2 & -2 \end{bmatrix}$. Find A^2.

$$A^2 = \begin{bmatrix} 2 & 2 \\ -2 & -2 \end{bmatrix} \begin{bmatrix} 2 & 2 \\ -2 & -2 \end{bmatrix} = \begin{bmatrix} (4-4) & (4-4) \\ (-4+4) & (-4+4) \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

5. Use matrix row reduction to solve: $\begin{cases} 2x+3y=-1 \\ 6x+11y=3 \end{cases}$

$$\left[\begin{array}{cc|c} 2 & 3 & -1 \\ 6 & 11 & 3 \end{array}\right] \xrightarrow{-3R_1+R_2} \left[\begin{array}{cc|c} 2 & 3 & -1 \\ 0 & 2 & 6 \end{array}\right] \xrightarrow{\frac{1}{2}R_2} \left[\begin{array}{cc|c} 2 & 3 & -1 \\ 0 & 1 & 3 \end{array}\right] \xrightarrow{R_1-3R_2} \left[\begin{array}{cc|c} 2 & 0 & -10 \\ 0 & 1 & 3 \end{array}\right] \xrightarrow{\frac{1}{2}R_1} \left[\begin{array}{cc|c} 1 & 0 & -5 \\ 0 & 1 & 3 \end{array}\right]$$

Therfore, the solution is:$x=-5, y=3$.

Check: $2(-5)+3(3) = -10+9 = -1$ and $6(-5)+11(3) = -30+33 = 3$

6. Use matrix row reduction to solve: $\begin{cases} x+3y+z=3 \\ 3x+8y+3z=7 \\ 2x-3y+z=-10 \end{cases}$

We will solve this system by changing to matrix form and transforming the matrix form of this system:

$$\left[\begin{array}{ccc|c} 1 & 3 & 1 & 3 \\ 3 & 8 & 3 & 7 \\ 2 & -3 & 1 & -10 \end{array}\right] \xrightarrow{R_2-3R_1} \left[\begin{array}{ccc|c} 1 & 3 & 1 & 3 \\ 0 & -1 & 0 & -2 \\ 2 & -3 & 1 & -10 \end{array}\right] \xrightarrow{R_3-2R_1} \left[\begin{array}{ccc|c} 1 & 3 & 1 & 3 \\ 0 & -1 & 0 & -2 \\ 0 & -9 & -1 & -16 \end{array}\right]$$

$$\xrightarrow{R_1+3R_2} \left[\begin{array}{ccc|c} 1 & 0 & 1 & 3 \\ 0 & -1 & 0 & -2 \\ 0 & -9 & -1 & -16 \end{array}\right] \xrightarrow{-R_2} \left[\begin{array}{ccc|c} 1 & 0 & 1 & 3 \\ 0 & 1 & 0 & 2 \\ 0 & -9 & -1 & -16 \end{array}\right] \xrightarrow{R_3+9R_2} \left[\begin{array}{ccc|c} 1 & 0 & 1 & -3 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & -2 \end{array}\right]$$

$$\xrightarrow{R_1-R_3} \left[\begin{array}{ccc|c} 1 & 0 & & -1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & -2 \end{array}\right]$$

Therefore, $x=-1, y=2$, and $z=-2$ is the unique solution to this system of linear equations.

7. Given the matrix: $A = \begin{bmatrix} 11 & 3 \\ 7 & 2 \end{bmatrix}$, find A^{-1}, the inverse of A

Then the augmented matrix is: $\left[\begin{array}{cc|cc} 11 & 3 & 1 & 0 \\ 7 & 2 & 0 & 1 \end{array}\right]$

We now reduce using row operations: $\left[\begin{array}{cc|cc} 11 & 3 & 1 & 0 \\ 7 & 2 & 0 & 1 \end{array}\right] \xrightarrow{2R_1-3R_2} \left[\begin{array}{cc|cc} 1 & 0 & 2 & -3 \\ 7 & 2 & 0 & 1 \end{array}\right] \xrightarrow{R_2-7R_1} \left[\begin{array}{cc|cc} 1 & 0 & 2 & -3 \\ 0 & 2 & -14 & 22 \end{array}\right]$

$\xrightarrow{\frac{1}{2}R_2} \left[\begin{array}{cc|cc} 1 & 0 & 2 & -3 \\ 0 & 1 & -7 & 11 \end{array}\right]$

Therefore, $A^{-1} = \begin{bmatrix} 2 & -3 \\ -7 & 11 \end{bmatrix}$

Check: $\begin{bmatrix} 11 & 3 \\ 7 & 2 \end{bmatrix} \begin{bmatrix} 2 & -3 \\ -7 & 11 \end{bmatrix} = \begin{bmatrix} (22-21) & (-33+33) \\ (14-14) & (-21+22) \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

8. Use the inverse matrix A^{-1} you found above to solve the system of equations: $\begin{cases} 11x + 3y = -5 \\ 7x + 2y = 1 \end{cases}$

   Translating this system to matrix form $AX = B$ gives: $\begin{bmatrix} 11 & 3 \\ 7 & 2 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -5 \\ 1 \end{bmatrix}$

   Then, $X = A^{-1}B$, or $\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 & -3 \\ -7 & 11 \end{bmatrix} \begin{bmatrix} -5 \\ 1 \end{bmatrix} = \begin{bmatrix} -10-3 \\ 35+11 \end{bmatrix} = \begin{bmatrix} -13 \\ 46 \end{bmatrix}$

   Check: $11(-13) + 3(46) = -143 + 138 = -5$ and $7(-13) + 2(46) = -91 + 92 = 1$

9. Given the matrix: $A = \begin{bmatrix} 4 & 2 & 2 \\ -1 & -3 & 4 \\ 3 & -1 & 6 \end{bmatrix}$, find A^{-1}, the inverse of A

   We now reduce using row operations: $\left[\begin{array}{ccc|ccc} 4 & 2 & 2 & 1 & 0 & 0 \\ -1 & -3 & 4 & 0 & 1 & 0 \\ 3 & -1 & 6 & 0 & 0 & 1 \end{array}\right] \xrightarrow{-R_2 \leftrightarrow R_1} \left[\begin{array}{ccc|ccc} 1 & 3 & -4 & 0 & -1 & 0 \\ 4 & 2 & 2 & 1 & 0 & 0 \\ 3 & -1 & 6 & 0 & 0 & 1 \end{array}\right]$

   $\xrightarrow[R_3 - 3R_1]{R_2 - 4R_1} \left[\begin{array}{ccc|ccc} 1 & 3 & -4 & 0 & -1 & 0 \\ 0 & -10 & 18 & 1 & 4 & 0 \\ 0 & -10 & 18 & 0 & 3 & 1 \end{array}\right] \xrightarrow{R_2 - R_3} \left[\begin{array}{ccc|ccc} 1 & 3 & -4 & 0 & -1 & 0 \\ 0 & -10 & 18 & 1 & 4 & 0 \\ 0 & 0 & 0 & 1 & 1 & -1 \end{array}\right]$

   Since we reached a matrix that has a row whose left side is all zeros, this matrix has no inverse.

10. Use the inverse matrix A^{-1} you found above to solve the system of equations: $\begin{cases} 4x + 2y + 2z = -3 \\ -x - 3y + 4z = 7 \\ 3x - y + 6z = 2 \end{cases}$

    Since A has no inverse function, we cannot solve this system of equations using the inverse matrix method.