Math 229
Optimization Applications
06/16/2008

The basic steps to solve optimization problems are:

a) Read the problem and organize the key facts and quantities described.

b) Draw and label a diagram representing the situation and introduce variables for unknown quantities.

c) Express all known facts and any relationships between the variables introduced.

d) Determine which variable is to be maximized or minimized and express it as a function of the other variables.

e) Differentiate the function formulated above and find its critical numbers.

f) Find and classify all extrema using the first or second derivative test. Also check endpoints for extrema if appropriate.

Solve the following optimization exercises:

1. Find the real number, which, when added to its square, gives the smallest sum.

2. A real estate company owns 180 apartments which are fully occupied when the rent is $300 per month. The company estimates that for each $10 increase in rent, five apartments will become unoccupied. What rent should be charged in order to obtain the largest gross income?

3. An offshore oil well is located in the ocean at a point W, 5 miles from the closest shore point A on a straight shoreline (See picture below). The oil must be piped to a shore point B that is 8 miles from A by piping it on a straight line under water from W to some shore point P between A and B and then on to B via a pipe along the shoreline. If the cost of laying pipe is $100,000 per mile under water and $75,000 per mile over land, where should point P be located in order to minimize the cost of laying the pipe? (round to the nearest hundredth of a mile) What will the minimum cost be?
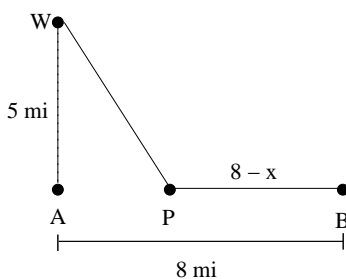
4. A concessionaire had been selling 5000 hot dogs at each football game at 50 cents apiece. When he raised the price to 70 cents apiece, sales dropped to only 4000 per night. Assume a linear relationship between price and sales. If the fixed costs each night are $1000, and each hot dog costs him 25 cents, what price per hot dog will maximize his nightly profit?