Math 127 - Exam 1 Solutions

1. (2 points) Express the number 123,782.34 in scientific notation.

   $= 1.2378234 \times 10^5$

2. (2 points) Rewrite the expression $|2\pi - 10|$ without using the absolute value symbol.

   Since $10 > 2\pi$, $|2\pi - 10| = 10 - 2\pi$

3. (2 points each) True or False:

   (a) $(a+b)^2 = a^2 + b^2$

   False

   Notice that $(a+b)^2 = (a+b)(a+b) = a^2 + 2ab + b^2$

   (b) $\dfrac{ad+bd}{cd} = \dfrac{a+b}{c}$

   True

   Notice that $\dfrac{ad+bd}{cd} = \dfrac{d(a+b)}{cd} = \dfrac{a+b}{c}$

   (c) $a^{\frac{1}{4}} = \dfrac{1}{a^4}$

   False

   Notice that $a^{\frac{1}{4}} = \sqrt[4]{a}$ while $\dfrac{1}{a^4} = a^{-4}$

   (d) $a \div \dfrac{b}{c} = \dfrac{ac}{b}$

   True

   Notice that $a \div \dfrac{b}{c} = \dfrac{a}{1} \cdot \dfrac{c}{b} = \dfrac{ac}{b}$

   (e) $\sqrt{a^2} = a$

   False

   Notice that if $a = -3$, $\sqrt{a^2} = \sqrt{-3^3} = \sqrt{9} = 3 \neq a$

4. (3 points each) Calculate each of the following. Express your answers in simplest form.

   (a) $\dfrac{11}{20} + \dfrac{5}{12}$

   $\dfrac{11}{20} \cdot \dfrac{3}{3} + \dfrac{5}{12} \cdot \dfrac{5}{5} = \dfrac{33}{60} + \dfrac{25}{60} = \dfrac{58}{60} = \dfrac{29}{30}$

   (b) $\dfrac{15}{28} \div \dfrac{25}{8}$

   $\dfrac{15}{28} \cdot \dfrac{8}{25} = \dfrac{5 \cdot 3}{4 \cdot 7} \cdot \dfrac{4 \cdot 2}{5 \cdot 5} = \dfrac{3 \cdot 2}{7 \cdot 5} = \dfrac{6}{35}$

5. Use properties of exponents and radicals to simplify the following expression. Your answer should have no negative exponents. Assume all variables represent nonnegative numbers.

(a) (5 points) $\left(x^{\frac{3}{5}}\right)^{\frac{5}{6}} \cdot x^{\frac{1}{2}}$

$= x^{\frac{3}{5}\cdot\frac{5}{6}} \cdot x^{\frac{1}{2}} = x^{\frac{1}{2}} \cdot x^{\frac{1}{2}} = x$

(b) (5 points) $\left(\dfrac{4x^2y^3}{12x^3y^{-5}}\right)^4$

$= \left(\dfrac{4x^2y^3y^5}{12x^3}\right)^4 = \left(\dfrac{y^8}{3x}\right)^4 = \dfrac{y^{32}}{81x^4}$

(c) (5 points) $\sqrt[3]{24x^8y^{10}}$

$= \sqrt[3]{8\cdot 3x^6x^2y^9y} = 2x^2y^3\sqrt[3]{3x^2y}$

6. Rationalize all denominators and simplify. Assume all variables represent positive values.

(a) (5 points) $\dfrac{5}{\sqrt[3]{2y^2}}$

$\dfrac{5}{\sqrt[3]{2y^2}} \cdot \dfrac{\sqrt[3]{4y}}{\sqrt[3]{4y}} = \dfrac{5\sqrt[3]{4y}}{2y}$

(b) (5 points) $\dfrac{3-\sqrt{x}}{\sqrt{x}+\sqrt{5}}$

$= \dfrac{3-\sqrt{x}}{\sqrt{x}+\sqrt{5}} \cdot \dfrac{\sqrt{x}-\sqrt{5}}{\sqrt{x}-\sqrt{5}} = \dfrac{3\sqrt{x}-3\sqrt{5}-x+\sqrt{5x}}{x-5}$

7. (5 points) Simplify the following expression:

$(3x-2y)^2-(3x+2y)^2$

$= 9x^2-6xy-6xy-(9x^2+6xy+6xy+4y^2) = 9x^2-12xy-9x^2-12xy-4y^2 = -24xy$

8. (5 points each) Factor each of the following *completely.* Box your answers.

(a) $4x^2 - 12x + 9 = (2x-3)(2x-3)$

(b) $x^3 + 5x^2 - 9x - 45 = x^2(x+5) - 9(x+5) = (x^2-9)(x+5) = (x+3)(x-3)(x+5)$

(c) $16x^4 - 81 = (4x^2+9)(4x^2-9) = (4x^2+9)(2x+3)(2x-3)$

(d) $8x^3 + 1 = (2x+1)(4x^2-2x+1)$ [Sum of cubes formula]

9. (7 points each) Perform the operations indicated and simplify each of the following as much as possible. Your answer should be completely reduced and should contain no complex fractions.

(a) $\dfrac{x+4}{x^2+3x-10} \div \dfrac{x^2+7x+12}{x^2+8x+15}$

$$= \frac{x+4}{(x+5)(x-2)} \div \frac{(x+4)(x+3)}{(x+5)(x+3)}$$

$$= \frac{x+4}{(x+5)(x-2)} \cdot \frac{(x+5)(x+3)}{(x+4)(x+3)} = \frac{1}{(x-2)}$$

(b) $\dfrac{4x}{x^2+5x+6} - \dfrac{3x}{x^2+x-2}$

$= \dfrac{4x}{(x+3)(x+2)} - \dfrac{3x}{(x+2)(x-1)} = \dfrac{4x(x-1)}{(x+3)(x+2)(x-1)} - \dfrac{3x(x+3)}{(x+3)(x+2)(x-1)}$

$= \dfrac{4x^2-4x}{(x+3)(x+2)(x-1)} - \dfrac{3x^2+9x}{(x+3)(x+2)(x-1)} = \dfrac{4x^2-4x-3x^2-9x}{(x+3)(x+2)(x-1)} = \dfrac{x^2-13x}{(x+3)(x+2)(x-1)}$

$= \dfrac{x(x-13)}{(x+3)(x+2)(x-1)}$

(c) $\dfrac{\frac{1}{x}+\frac{3}{x-2}}{\frac{4}{x-1}-\frac{2}{x-2}}$

$= \dfrac{\frac{x-2}{x(x-2)}+\frac{3x}{x(x-2)}}{\frac{4(x-2)}{(x-1)(x-2)}-\frac{2(x-1)}{(x-1)(x-2)}} = \dfrac{\frac{x-2+3x}{x(x-2)}}{\frac{4x-8-2x+2}{(x-1)(x-2)}} = \dfrac{\frac{4x-2}{x(x-2)}}{\frac{2x-6}{(x-1)(x-2)}}$

$= \dfrac{4x-2}{x(x-2)} \cdot \dfrac{(x-1)(x-2)}{2x-6} = \dfrac{2(2x-1)(x-1)}{x(2)(x-3)} = \dfrac{(2x-1)(x-1)}{x(x-3)}$

10. Solve each of the following equations:

(a) (5 points) $(3x-1)^2 = (x+2)(9x+1)$

$9x^2 - 3x - 3x + 1 = 9x^2 + 18x + x + 2$

$-6x + 1 = 19x + 2$

$1 - 2 = 19x + 6x$

$-1 = 25x$

$x = -\frac{1}{25}$

(b) (5 points) $\dfrac{2x-1}{x+1} - 3 = 0$

$\dfrac{2x-1}{x+1} = 3$

$(x+1) \cdot \left[\dfrac{2x-1}{x+1}\right] = 3 \cdot (x+1)$

$2x - 1 = 3x + 3$

$-1 - 3 = 3x - 2x$

Thus $x = -4$

11. (5 points) Solve for v in the equation: $sv = \frac{1}{3}xy + 2vc$

$sv - 2vc = \frac{1}{3}xy$, or $v(s-2c) = \frac{1}{3}xy$

Therefore, $v = \frac{\frac{1}{3}xy}{s-2c} = \frac{xy}{3(s-2c)}$