Math 127 - College Algebra

Handout: Algebraic Expressions, Sets, Mathematical Models, and Real Numbers

• An **algebraic expression** is a combination of variables and numbers using the operations of addition, subtraction, multiplication or division, as well as powers or roots.

Examples:
$$3x$$
, $2x - y$, $x^2 - 4xy$, $\sqrt{2x} + 7$

• Evaluating an algebraic expression is finding the value of the expression for given values for each of the variables represented in the expression.

Note: Use "Order of Operations" [PEMDAS] when evaluating any algebraic expression.

- 1. Perform operations within grouping symbols like parentheses, brackets, and braces beginning within the innermost grouping and working outwards.
- 2. Evaluate all exponential expression before other operations.
- 3. Perform multiplications and divisions, working from left to right.
- 4. Perform additions and subtractions, working from left to right.

Examples:

- 1. Evaluate $\sqrt{2x} + 7$ when x = 8
- 2. Evaluate $x^2 4xy$ when x = 3 and y = -2
- 3. Evaluate $2x^2 7(x-3)^3$ when x = 2
- A set is a collection of objects of some type. We usually use capital letters to denote sets. Individual elements are called **elements**.

Examples:
$$\mathbb{N} = \{1, 2, 3, ...\}$$
, $\mathbb{W} = \{0, 1, 2, 3, ...\}$, $\mathbb{Z} = \{... -2, -1, 0, 1, 2, 3, ...\}$, $\{x \mid x^2 < 16 \text{ and } x \text{ is an integer }\}$

Notation:

- $A \cap B$ is the **intersection** of the sets A and B. That is, all of the elements that the two sets have in common.
- $A \cup B$ is the **union** of the sets A and B. That is, all of the elements that are in either of the two sets.
- \emptyset denotes the empty set, a set that has no elements in it.

Examples:

1.
$$\{2,4,7,9\} \cup \{1,4,5,7\} = \{1,2,4,5,7,9\}$$

2.
$$\{2,4,7,9\} \cap \{1,4,5,7\} = \{4,7\}$$

3.
$$\{2,3,5,7\} \cap \{1,6,8,9\} = \emptyset$$

• Recall the definitions of Natural Numbers, Whole Numbers, Integers, Rational Numbers, Irrational Numbers, and Real Numbers [See page 7 in your textbook]

Given the set: $\{-3, 0, \frac{1}{2}, \frac{16}{2}, \sqrt{2}, -\frac{7}{2}, .314159, \sqrt{16}, \sqrt{-1}, \pi\}$:

- 1. Which elements of this set are integers?
- 2. Which elements of this set are rational?
- 3. Which elements of this set are irrational?
- The symbols: $<, \le, >, \ge$ arise from the standard ordering of the real line and allow us to indicate the relative size of any pair of real numbers.
- Absolute Value:

$$|a| = \begin{cases} a & \text{if } a \ge 0\\ -a & \text{if } a < 0 \end{cases}$$

Examples: (a) |7| = 7

(b)
$$|-4|=4$$

Properties:

1.
$$|a| \ge 0$$

$$|a| - |a| = |a|$$

3.
$$|ab| = |a||b|$$

4.
$$\left| \frac{a}{b} \right| = \frac{|a|}{|b|}$$

5.
$$|a+b| \le |a| + |b|$$

Evaluating Absolute Value Expressions:

$$(i)|\pi - 3| = \pi - 3$$

$$(ii)|3-\pi| = \pi - 3$$

$$(iii)\frac{|-4|-|7|}{|-4-7|} = ?$$

- Review the Properties of Real Numbers on pages 12-13 in your textbook.
- Using the Properties of Real Numbers and Order of Operations, we can simplify many algebraic expressions.

Example: Simplify the algebraic expression $7x^2 - 3 + [3(x^2 - 1) - 10]$