

Math 127 - College Algebra

Handout: Equations

A. Definitions

- An **equation** is a statement that two algebraic expressions are equal. Equations can involve one or more variables, constants, and mathematical operations.
- An equation **in x** is one that involves *only one variable*, namely, x .

Example: $x^3 + \sqrt{x} = 14x$

- A **root** or **solution** to an equation is a value that, when substituted for x into the equation, makes the equation true.

Example: $x = 0$ is a solution to the equation $x^3 + \sqrt{x} = 14x$ [Check this!]
Similarly, $x = 1$ is *not* a solution to this equation [Check this too!]

- Two equations that have the same **solution set** are called **equivalent**.
- To **solve** an equation is to find its solution set (i.e. *all* of its solutions).
- There are three main types of equations:
 1. **Identities** are *always* true (i.e. every number is in the solution set).
 2. **Conditional equations** are *sometimes* true.
 3. **Inconsistent equations** are *never* true (i.e. the solution set is \emptyset)

Examples:

1. $3x - x = x + x$ is an identity equation.
2. $x^3 + \sqrt{x} = 14x$ is a conditional equation.
3. $x + 3 = x + 2$ is an inconsistent equation.

Notes:

- To solve an equation, we generally apply mathematical operations to the equation that result in an equivalent equation that is easier to understand so that we can “read off” the solution set.
- Sometimes we will use mathematical operations that are useful in understanding the solution set of an equation but that *do not* result in an equivalent equation [i.e. they may introduce *extraneous solutions*]. We need to be especially careful to check our answers in situations where these methods are used.

Warning: The methods we use to solve equations are significantly different from those that we use to simplify algebraic expressions. That is due to the fact that equations have “two sides” while expressions only have “one side”. It is important that you think about what you are doing and that you do not “mix and match” your methods.

B. Linear Equations

- A **linear equation in one variable** is any equation that can be put into the form $ax + b = 0$
- To solve linear equations, we usually *isolate the variable* (i.e. get it by itself on one side of the equation).
- Two operations that are useful in solving equations are:
 - Adding/Subtracting a term to/from both sides of an equation.
 - Multiplying/Dividing both sides of an equation by a **non-zero** constant.

Example:

$$\begin{array}{rcl} 10x + 3 & = & 0 \\ -3 & & -3 \\ \hline 10x & = & -3 \\ \div 10 & & \div 10 \\ x & = & -\frac{3}{10} \end{array}$$

Note: As we move forward, we will look at more complicated equations and we will use these and other techniques in order to find their solution sets.