Math 127
Exam 4 Practice Problems

1. Solve the following systems of linear equations. Then graph the equations involved to show that your answer is reasonable.

   (a) $\begin{cases} y = 3x - 2 \\ x + 2y = 5 \end{cases}$

   (b) $\begin{cases} 3x - 2y = 7 \\ x + 2y = 5 \end{cases}$

   (c) $\begin{cases} x - 3y = 2 \\ 6y - 2x = 5 \end{cases}$

   (d) $\begin{cases} 5x - 4y = 10 \\ 3x + 5y = 12 \end{cases}$

   (e) $\begin{cases} x - 2y = 5 \\ -2x + 4y = 10 \end{cases}$

2. Graph each of the following functions:

   (a) $f(x) = 4^x$
   (b) $f(x) = 5^{-x}$
   (c) $f(x) = 2^x - 1$
   (d) $f(x) = 2^{x-1}$

3. Solve the following equations:

   (a) $3^{2-3x} = 3^{2x+1}$
   (b) $5^{4x} = 5^{3x-12}$
   (c) $2^{5x+1} = 4^{3-2x}$

4. Suppose you have \$2,000 to invest.

   (a) Find the amount you would have after 5 years if you deposit your \$2,000 in an account that pays 6% annual interest compounded monthly.
   (b) Find the amount you would have after 5 years if you deposit your \$2,000 in an account that pays 4% annual interest compounded quarterly.
   (c) Find the amount you would have after 5 years if you deposit your \$2,000 in an account that pays 5% annual interest compounded continuously.

5. Translate each of the following expressions into exponential form:

   (a) $\log_5 x = y$
   (b) $\log_z 5 = y$
   (c) $\log_y x = 5$

6. Find the *exact* value of each of the following:

   (a) $\log_{21}(1)$
   (b) $\log_7(0)$
   (c) $\log_2(\frac{1}{8})$
   (d) $\log_3(27)$
   (e) $\ln(e^2)$
   (f) $\log .0001$
   (g) $\log_9(27)$
   (h) $7^{\log_7(\pi)}$

7. Determine whether the following are True or False:

   (a) $\ln\left(\frac{x^3}{(x+1)(x-1)}\right) = 3\ln x - \ln(x+1) + \ln(x-1)$

(b) $e^{\ln(x^2+1)} = x^2 + 1$

(c) $e^{x^2} \cdot e^{3x} = e^{3x^3}$

(d) $\dfrac{\ln(4x)}{\ln(2x)} = \ln 2$

(e) $\ln\left(e^{x^2} - 4\right) = x^2 - \ln 4$

8. Use properties of logarithms to expand the following expression:
$\log\left(\dfrac{x^4 z^2}{\sqrt[3]{y}}\right)$

9. Use the laws of logarithms to expand the expression: $\ln\left(\dfrac{x^2(x-1)^{\frac{5}{2}}}{(x-4)^3}\right)$

10. Use the properties of logarithms to write the following as a single logarithm:
$\dfrac{3}{2}\log_b x^3y^4 - \dfrac{2}{3}\log_b x^4y^3 - 2\log_b xy$

11. Use the change of base formula to approximate the following:

    (a) $\log_5 10$
    (b) $\log_9 12$
    (c) $\log_{15} 7$

12. Convert the following functions to base e exponentials:

    (a) $f(x) = 100 \cdot 4^x$
    (b) $g(x) = 50 \cdot \left(\frac{1}{4}\right)^x$
    (c) $h(x) = 450 \cdot 5^{-2x}$

13. Solve the following equations (give exact answers whenever possible):

    (a) $e^{3x-2} = e^{4-5x}$
    (b) $9^{2x} = 27(3)^{2x+1}$
    (c) $\log_2(3x^2 - 3) = \log_2(x^2 + x)$
    (d) $\log_5(x^2 + 21) = 2$
    (e) $\log_2(2x) + \log_2(x-3) = 3$
    (f) $\log(\sqrt[4]{x+1}) = \dfrac{1}{2}$
    (g) $e^{2x-1} = 3$
    (h) $4^{2x-1} = 3^{5x}$

14. (a) Suppose you invest \$10,000 in a savings account that pays 3% annual interest compounded monthly. How much money will be in the account after 6 years?
    (b) How long would it take \$5,000 invested at 6% annual interest compounded continuously to triple?
    (c) Find the interest rate needed for an investment of \$2,000 to double in 6 years if the interest is compounded quarterly.

15. Suppose that a culture of bacteria that initially has 500 cells grows to 10,000 cells in 12 hours.

    (a) Find a function $f(t)$ that gives the number of cells in the culture as a function of time (in hours), assuming that this population grows continuously and exponentially.
    (b) How long will it take for the culture to reach 1,000,000 cells?

16. Suppose a certain substance has a half life of 47 years. If you start with 100 grams of the substance, how long will it take for the amount to be reduced to 50 grams? How long will it take for the amount to be reduced to 12 grams?