

Math 127 - College Algebra  
Handout: Properties of Exponents and Radicals

**A. Exponents**

**Definition:**  $a^n = a \cdot a \cdot a \cdot a \cdot \dots \cdot a$  ( $a$  multiplied by itself  $n$  times)

**Properties:**

1.  $a^0 = 1$
2.  $a^{-n} = \frac{1}{a^n}$
3.  $a^m \cdot a^n = a^{m+n}$
4.  $(a^m)^n = a^{mn}$
5.  $(ab)^n = a^n b^n$
6.  $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$
7.  $\frac{a^m}{a^n} = a^{m-n} = \frac{1}{a^{n-m}}$
8.  $\frac{a^{-m}}{b^{-n}} = \frac{b^n}{a^m}$
9.  $\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n$

**B. Radicals:**

**Definition:** Suppose  $n$  is a positive integer and  $a$  is a real number. Then we define the  **$n$ th root of  $a$** , denoted by  $\sqrt[n]{a}$  as follows:

- If  $a = 0$ , then  $\sqrt[n]{a} = 0$ .
- If  $a > 0$  then  $\sqrt[n]{a}$  is the *positive* real number  $b$  such that  $b^n = a$ .
- If  $a < 0$  and  $n$  is **odd**, then  $\sqrt[n]{a}$  is the *negative* real number  $b$  such that  $b^n = a$ .
- If  $a < 0$  and  $n$  is **even**, then  $\sqrt[n]{a}$  is not a real number, since there is no real number  $b$  such that  $b^n = a$ .

**Examples:**

(a)  $\sqrt[2]{9} = \sqrt{9} = 3$  since  $3 \cdot 3 = 9$ .                      (b)  $\sqrt[3]{-8} = -2$  since  $(-2) \cdot (-2) \cdot (-2) = -8$ .

(c)  $\sqrt{-16}$  is not a real number. (notice that  $4 \cdot 4 = 16$ , and  $(-4) \cdot (-4) = 16$ )

**Properties:**

1.  $(\sqrt[n]{a})^n = a$  if  $\sqrt[n]{a}$  is a real number.
2.  $\sqrt[n]{a^n} = a$  if  $a \geq 0$ .
3.  $\sqrt[n]{a^n} = a$  if  $a < 0$  and  $n$  is odd.
4.  $\sqrt[n]{a^n} = |a|$  if  $a < 0$  and  $n$  is even.
5.  $\sqrt[n]{ab} = \sqrt[n]{a} \sqrt[n]{b}$  provided both exist.
6.  $\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$  provided both exist.
7.  $\sqrt[m]{\sqrt[n]{a}} = \sqrt[mn]{a}$  provided both exist.

**Warning!!**

(a) In general,  $\sqrt{a^2 + b^2} \neq a + b$

(b) Also, in general,  $\sqrt{a+b} \neq \sqrt{a} + \sqrt{b}$

**Exponents and Radicals:**

1.  $\sqrt[n]{a} = a^{\frac{1}{n}}$ .
2.  $\sqrt[n]{a^m} = a^{\frac{m}{n}} = \left(a^{\frac{1}{n}}\right)^m = (a^m)^{\frac{1}{n}}$ .