Math 127
Logarithms

Definition: The **Logarithm of** x **to the base** b is defined as follows: $y = \log_b x$ if and only if $x = b^y$. for $x > 0$ and $b > 0, b \neq 1$. A logarithm basically asks: "what power would I need to raise the base b to in order to get x as the result?"

Examples:

Logarithmic Form:	Exponential Form:
(a) $\log_2 8 = 3$	$8 = 2^3$
(b) $\log_2 \frac{1}{2} = -1$	$\frac{1}{2} = 2^{-1}$
(c) $\log_3 81 = 4$	$81 = 3^4$
(d) $\log_8 \frac{1}{64} = -2$	$\frac{1}{64} = 8^{-2}$
(e) $\log_2 -8$ is undefined	$2^y \neq -8$ for any possible y!

Solving Logarithmic Equations:

Note: Since logarithmic functions are inverses of exponential functions, logarithmic functions are one-to-one. Therefore, as before, we can make use of the definition of a one-to-one function in order to solve basic equations involving logarithmic functions.

Warning!! Since $\log_b x$ is only defined for $x > 0$, we will need to check for extraneous solutions. Any value that makes the expression inside a logarithm negative is not a valid solution.

Examples:

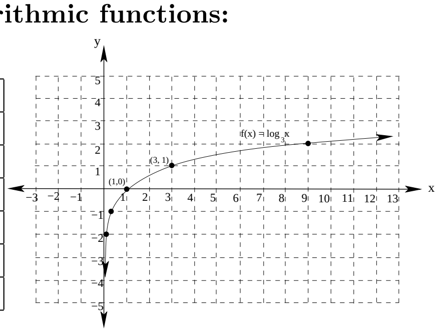
(a) Suppose $\log_5 x = 3$. Find x.
Since $\log_5 x = 3$, $x = 5^3 = 125$.
(b) Suppose $\log_z 16 = 2$. Find z.
Since $\log_z 16 = 2$, $z^2 = 16$, so $z = \pm 4$. But since we know that $z > 0$, then $z = 4$.
(c) $\log_4(3x + 1) = \log_4(2x + 4)$
Since $\log_4 x$ is one-to-one, we know $3x + 1 = 2x + 4$
Therefore, $x = 3$.
(d) $\log_7(x^2 + 8x) = \log_7(10x + 8)$

Notation: If $b = 10$, we abbreviate $\log_{10} x$ as $\log x$. Similarly, if $b = e$, we abbreviate $\log_e x$ as $\ln x$.

Graphs of logarithmic functions:

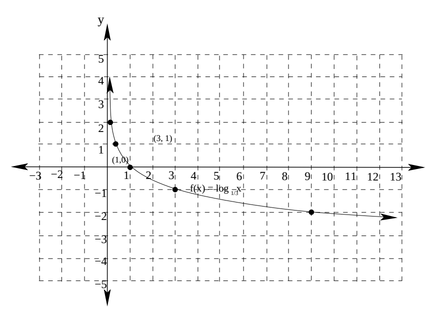
$f(x) = \log_3 x$

x	$f(x)$
0	undefined
1	0
3	1
9	2
$\frac{1}{3}$	-1
$\frac{1}{9}$	-2



$f(x) = \log_{\frac{1}{3}} x$

x	$f(x)$
0	undefined
1	0
$\frac{1}{3}$	1
$\frac{1}{9}$	2
3	-1
9	-2



Properties of Logarithmic Graphs:

1. Domain: $(0, \infty)$
2. Range: $(-\infty, \infty)$
3. y-intercept: none. x intercept $(1, 0)$
4. Increasing if $b > 1$. Decreasing if $0 < b < 1$.

Examples: Finding doubling times using logarithms.
(a)

(b)