Math 127
Polynomial Practice Problem Solutions

1. For each quadratic function given below, find the coordinates of the vertex, and find the equation for the line of symmetry for the graph of the function.

   (a) $f(x) = (x-5)^2 + 12$

   Since this quadratic function is already in vertex form, we can see from the form of the equation that the vertex is $(5, 12)$ and the axis of symmetry is at $x = 5$.

   (b) $f(x) = (x+3)^2 - 8$

   Since this quadratic function is already in vertex form, we can see from the form of the equation that the vertex is $(-3, -8)$ and the axis of symmetry is at $x = -3$.

   (c) $f(x) = 2x^2 - 12x + 22$

   This quadratic function is not in vertex form, so we can either use completing the square to put it in vertex form, or, instead, we can just recall that $h = \frac{-b}{2a} = \frac{12}{4} = 3$. Then $k = f(h) = f(3) = 2(3)^2 - 12(3) + 22 = 18 - 36 + 22 = 4$. Then the vertex is $(3, 4)$ and the axis of symmetry is at $x = 3$.

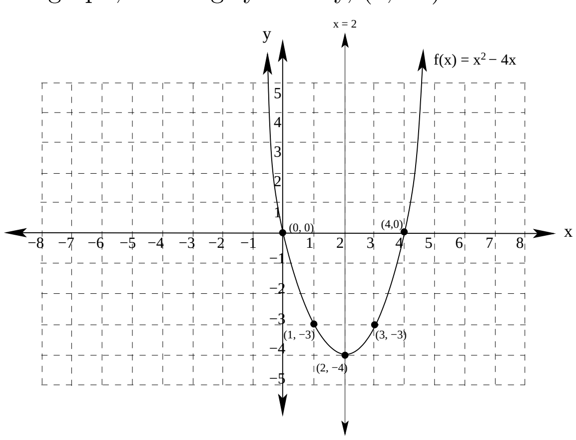
   (d) $f(x) = -4x^2 + 16x - 13$

   This quadratic function is not in vertex form, so we can either use completing the square to put it in vertex form, or, instead, we can just recall that $h = \frac{-b}{2a} = \frac{-16}{-8} = 2$. Then $k = f(h) = f(2) = -4(2)^2 + 16(2) - 13 = -16 + 32 - 13 = 3$. Then the vertex is $(2, 3)$ and the axis of symmetry is at $x = 2$.

2. For each quadratic function given below, find the vertex, line of symmetry, and (if applicable) the x-intercepts of the function. Then graph the function carefully in the xy-plane.

   (a) $f(x) = x^2 - 4x$

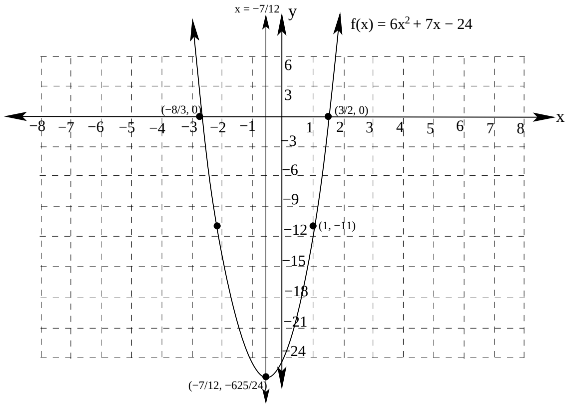
   Since the form of this quadratic is fairly simple, we will go ahead and complete the square to put it into vertex form:
   $f(x) = (x^2 - 4x + 4) - 4 = (x-2)^2 - 4)$. So the vertex is $V : (2, -4)$ and the axis of symmetry is $x = 2$.
   Next, we find the intercepts: $f(0) = 0$, so $(0, 0)$ is the y-intercept [notice that this is also an x-intercept]
   To find the x-intercepts, we factor $x^2 - 4x = 0$ to get $x(x-4) = 0$, so the intercepts are at $x = 0$ and $x = 4$.
   Finally, we plot one more value to get a better idea of the shape of our graph: $f(1) = 1 - 4 = -3$, so $(1, -3)$ is on the graph, so using symmetry, $(3, -3)$ is also on the graph. Then we have the graph:



   (b) $f(x) = 6x^2 + 7x - 24$

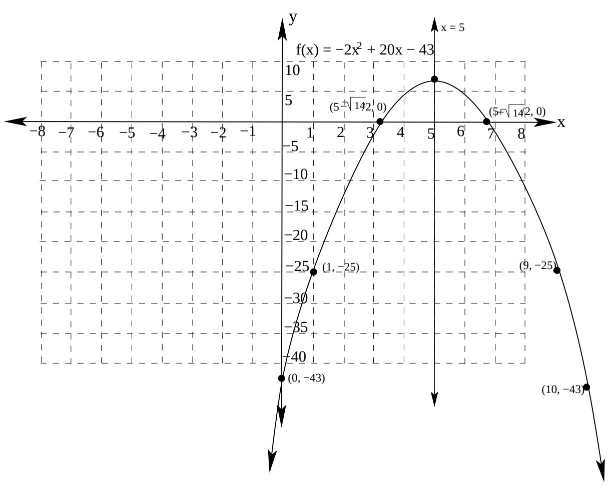
   Since the form of this quadratic is a bit messy, we will find the vertex algebraically:
   $h = \frac{-b}{2a} = -\frac{7}{12}$, so $k = f(h) = f\left(-\frac{7}{12}\right) = 6\left(-\frac{7}{12}\right)^2 + 7\left(-\frac{7}{12}\right) - 24 == -\frac{625}{24} \approx -26.042$
   So the vertex is $V : (-\frac{7}{12}, -\frac{625}{24})$ and the axis of symmetry is $x = -\frac{7}{12}$
   Next, we find the intercepts: $f(0) = -24$, so $(0, -24)$ is the y-intercept.
   To find the x-intercepts, we factor $6x^2 + 7x - 24 = 0$ to get $(3x+8)(2x-3) = 0$, so the intercepts are at $x = \frac{3}{2}$ and $x = -\frac{8}{3}$.

Finally, we plot one more value to get a better idea of the shape of our graph: $f(1) = 6+7-24 = -11$, so $(1,-11)$ is on the graph.



(c) $f(x) = -2x^2 + 20x - 43$

Since the form of this quadratic is a bit messy, we will find the vertex algebraically:
$h = \frac{-b}{2a} = \frac{-20}{-4} = 5$, so $k = f(h) = f(5) = -2(5)^2 + 20(5) - 43 = -50 + 100 - 43 = 7$
So the vertex is $V : (5,7)$ and the axis of symmetry is $x = 5$
Next, we find the intercepts: $f(0) = -43$, so $(0,-43)$ is the y-intercept.
To find the x-intercepts, we look at $-2x^2 + 20x - 43 = 0$. Since this does not factor, we use the quadratic equation to find $x = \frac{-20 \pm \sqrt{20^2 - 4(-2)(-43)}}{2(-2)} = \frac{-20 \pm \sqrt{400-344}}{-4} = 5 \pm \frac{\sqrt{56}}{-4} = 5 \pm \frac{\sqrt{14}}{2}$
Finally, we plot one more value to get a better idea of the shape of our graph: $f(1) = -2 + 20 - 43 = -25$, so $(1,-25)$ is on the graph.



3. For each polynomial function given below, use the degree of the polynomial and the sign of the leading coefficient to describe the end behavior of the polynomial.

(a) $f(x) = 5x^4 - 3x^2 + 7$

Since $5 > 0$ and the leading term $5x^4$ has even degree, both ends of this graph are moving in the positive direction.

(b) $f(x) = -2x^3 + 4x^2 - 15x + 12$

Since $-2 < 0$ and the leading term $-2x^3$ has odd degree, the left end of this graph is moving in the positive direction while the right end is moving in the negative direction.

(c) $f(x) = 12x^5 - 5x^4 + 3x^2 - 12x + 7$

Since $12 > 0$ and the leading term $12x^5$ has odd degree, the left end of this graph is moving in the negative direction while the right end is moving in the positive direction.

(d) $f(x) = -7x^6 + 5x^4 - 13x^3 + 21x^2 + 15x + 23$

Since $-7 < 0$ and the leading term $-7x^4$ has even degree, both ends of this graph are moving in the negative direction.

4. For each polynomial function given below, find the zeroes of the polynomial and also give the multiplicity of each zero.

   (a) $f(x) = x^4 + 3x^3 - 4x^2$

   Factoring, we have $f(x) = x^2(x^2 - 3x - 4) = x^2(x+4)(x-1)$.
   Therefore, the zeroes are $x = 0$, which has multiplicity 2, $x = -4$, which has multiplicity 1, and $x = 1$, which has multiplicity 1.

   (b) $f(x) = x^3 + 2x^2 - 4x - 8$

   Factoring by grouping, we have $f(x) = x^2(x+2) - 4(x+2) = (x^2-4)(x+2) = (x+2)(x-2)(x+2) = (x+2)^2(x-2)$.
   Therefore, the zeroes are $x = -2$, which has multiplicity 2 and $x = 2$, which has multiplicity 1.

   (c) $f(x) = x(x-4)^4(x+3)^3$

   Since this expression has already been factored, we can just read off the zeroes and their multiplicities:
   $x = 0$, which has multiplicity 1, $x = 4$, which has multiplicity 4, and $x = -3$, which has multiplicity 3.

   (d) $f(x) = x^3(x-2)^2(x^2-4)^5$

   This expression has been partially factored, be we cannot just read off the zeroes in this form, we we factor further to get $f(x) = x^3(x-2)^2(x^2-4)^5 = x^2(x-2)^2((x+2)(x-2))^5 = x^2(x-2)^2(x+2)^5(x-2)^5 = x^2(x-2)^7(x+2)^5$
   Then the zeroes are: $x = 0$, which has multiplicity 3, $x = 2$, which has multiplicity 7, and $x = -2$, which has multiplicity 5.

5. Use the intermediate value theorem to show that $f(x) = 2x^3 + 5x^2 - 3$ has a zero between $x = -3$ and $x = -2$

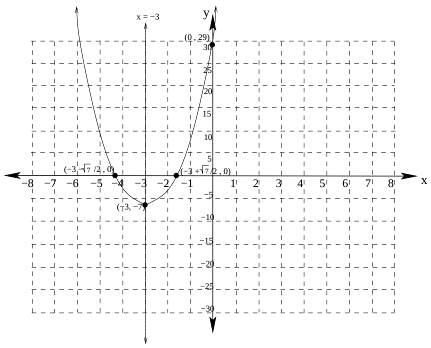
   Notice that $f(-3) = 2(-3)^3 + 5(-3)^2 - 3 = -54 + 45 - 3 = -12$
   While $f(-2) = 2(-2)^3 + 5(-2)^2 - 3 = -16 + 20 - 3 = 1$
   Since $f(-3)$ is negative and $f(-2)$ is positive, by the Intermediate Value Theorem, there must be a value c with $-3 < c < -2$ and $f(c) = 0$.

6. For each of the following quadratic functions:
   - Find the vertex and axis of symmetry
   - Find all intercepts
   - Graph the function and list its range in interval notation

   (a) $f(x) = 4(x+3)^2 - 7$

   Since this function is already in vertex form $f(x) = a(x-h)^2 + k$, we see that the vertex is $(-3, -7)$ and the axis of symmetry is the line $x = -3$.

   To find the intercepts, we multiply out to obtain: $f(x) = 4(x^2 + 6x + 9) - 7 = 4x^2 + 24x + 36 - 7 = 4x^2 + 24x + 29$
   Since this does not factor, we use the quadratic formula to find the x-intercepts:
   $$x = \frac{-24 \pm \sqrt{24^2 - 4(4)(29)}}{8} = \frac{-8 \pm 4\sqrt{7}}{8} = -3 \pm \frac{\sqrt{7}}{2}$$
   The y-intercept is $(0, 29)$. Then the graph is as shown below, and the range is $[-7, \infty)$.



(b) $f(x) = -3(x-4)^2 + 11$

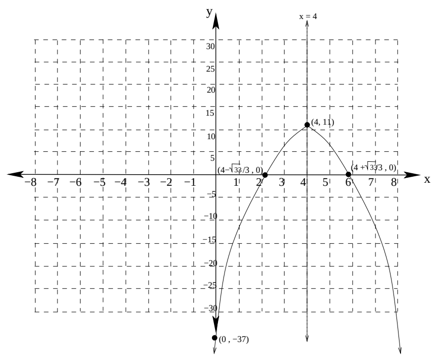
Since this function is already in vertex form $f(x) = a(x-h)^2 + k$, we see that the vertex is $(4, 11)$ and the axis of symmetry is the line $x = 4$.

To find the intercepts, we multiply out to obtain: $f(x) = -3(x^2 - 8x + 16) + 11 = -3x^2 + 24x - 48 + 11 = -3x^2 + 24x - 37$

Since this does not factor, we use the quadratic formula to find the x-intercepts:

$$x = \frac{-24 \pm \sqrt{24^2 - 4(-3)(37)}}{-6} = \frac{-24 \pm \sqrt{132}}{-6} = 4 \pm \frac{2\sqrt{33}}{-6} = 4 \pm \frac{\sqrt{33}}{3}$$

The y-intercept is $(0, -37)$. Then the graph is as shown below, and the range is $(-\infty, 11]$.



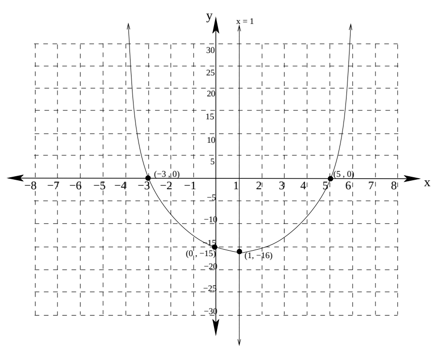
(c) $f(x) = x^2 - 2x - 15$

To find the vertex, we will make use of the formula $h = -\frac{b}{2a} = \frac{2}{2} = 1$. Also, $f(1) = 1 - 2 - 15 = -16$. Therefore, the vertex is: $(1, -16)$ and the axis of symmetry is the line $x = 1$.

To find the x-intercepts, we look at the equation: $x^2 - 2x - 15 = 0$, which factors to give $(x-5)(x+3) = 0$

Then the x intercepts occur when $x = 5$ and $x = -3$.

The y-intercept is $f(0) = -15$. Then the graph is as shown below, and the range is $[-16, \infty)$.



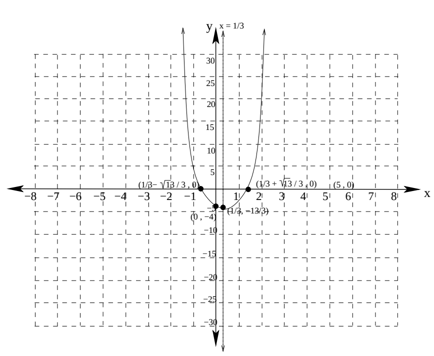
(d) $f(x) = 3x^2 - 2x - 4$

To find the vertex, we will make use of the formula $h = -\frac{b}{2a} = \frac{2}{6} = \frac{1}{3}$. Also, $f\left(\frac{1}{3}\right) = 3\left(\frac{1}{3}\right)^2 - 2\left(\frac{1}{3}\right) - 4 = \left(\frac{1}{3}\right) - \left(\frac{2}{3}\right) - \frac{12}{3} = -\frac{13}{3}$. Therefore, the vertex is: $\left(\frac{1}{3}, \frac{13}{3}\right)$ and the axis of symmetry is the line $x = \frac{1}{3}$.

To find the x-intercepts, since the equation does not factor, we use the quadratic formula:

$$x = \frac{2 \pm \sqrt{4 - 4(3)(-4)}}{6} = \frac{2 \pm \sqrt{52}}{6} = \frac{2 \pm 2\sqrt{13}}{6} = \frac{1}{3} \pm \frac{\sqrt{13}}{3}$$

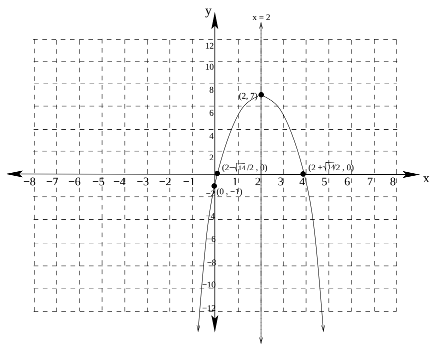
The y-intercept is $(0, -4)$. Then the graph is as shown below, and the range is $[-\frac{13}{3}, \infty)$.



(e) $f(x) = -2x^2 + 8x - 1$

To find the vertex, we will make use of the formula $h = -\frac{b}{2a} = \frac{8}{4} = 2$. Also, $f(2) = -8 + 16 - 1 = 7$ Therefore, the vertex is: $(2, 7)$ and the axis of symmetry is the line $x = 2$.

To find the x-intercepts, since the equation does not factor, we use the quadratic formula:
$x = \dfrac{-8 \pm \sqrt{64 - 4(-2)(-1)}}{-4} = 2 \pm \dfrac{\sqrt{56}}{-4} = 2 \pm \dfrac{2\sqrt{14}}{4} = 2 \pm \dfrac{\sqrt{14}}{2}$
The y-intercept is $(0, -1)$. Then the graph is as shown below, and the range is $(-\infty, 7]$.



7. Among all pairs of numbers whose sum is 20, find the pair whose product is as large as possible. Also find the value of the maximum product.

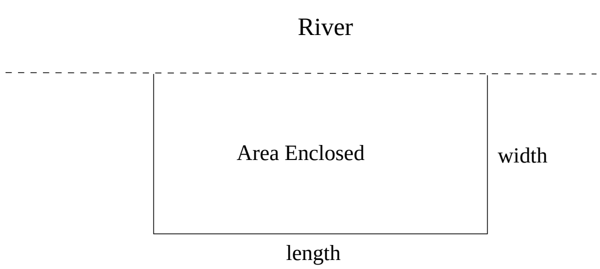
Let x be the first number and y the second. We know that the sum of these two numbers is 20, so $x + y = 20$. We want to find the minimum value of their product xy.

Since $x + y = 20$, then $y = 20 - x$. Substituting this, $xy = x(20 - x)$. Then we want to find the minimum value of the function $f(x) = x(20 - x)$. Multiplying this out, $f(x) = 20x - x^2$, or $f(x) = -x^2 + 20x$. Notice that this is a quadratic function. Therefore, since the coefficient of the x^2 term is negative, the graph of this function is a parabola that opens downward, and the minimum value occurs at the vertex.

To find the vertex, we could complete the square on this expression for the function, but we will instead use the formula $h = \frac{-b}{2a} = \frac{-20}{-2} = 10$. Then we must have $x = 10$, $y = 20 - 10 = 10$, and the maximum product is $f(10) = 20(10) - 10^2 = 200 - 100 = 100$.
That is, the two numbers are 10 and 10, and their product is 100.

8. Suppose that you have 600 feet of fencing to enclose a rectangular field that borders a river on one side. If you do not fence the side along the river, find the length and width of the maximum sized field that you can fence in.

We begin by drawing a diagram for this situation:



If we let x be the width of the field enclosed and y the length, then the sum of the three sides $x + y + x$ must equal the total amount of fencing material used: $2x + y = 600$. The area enclosed is given by $A = xy$.

If we solve for y in the first equation, $y = 600 - 2x$, then $f(x) = x(600 - 2x) = 600x - 2x^2$ gives the area of the field as s function of the width x.
The zeros of this function are at $x = 0$ and $600 = 2x$, or $x = 300$, so the maximum area is at the vertex, which is always halfway between the two zeros, or at $x = 150$.
Therefore, the dimensions of the field must be 150 by 300 feet.

9. Find the zeros for each polynomial function given. Also find the multiplicity of each zero.

   (a) $f(x) = x^3 - x^2 - 9x + 9$

   Using factoring by grouping, we have $f(x) = x^2(x-1) - 9(x-1)$, or $f(x) = (x^2-9)(x-1) = (x+3)(x-3)(x-1)$. Then the zeros are $x = -3$ which has multiplicity 1, $x = 3$ which has multiplicity 1, and $x = 1$ which has multiplicity 1.

   (b) $f(x) = 2x^3 - 5x^2 - 12x$

   We begin by factoring out the greatest common factor: $f(x) = x(2x^2 - 5x - 12)$. Then, using trinomial factoring (or the ac split if you prefer), $f(x) = x(2x+3)(x-4)$. Notice that if $2x + 3 = 0$, then $2x = -3$, so $x = -\frac{3}{2}$. Then the zeros are $x = 0$ which has multiplicity 1, $x = -\frac{3}{2}$ which has multiplicity 1, and $x = 4$ which has multiplicity 1.

   (c) $f(x) = x^2(x-1)(x+3)^3$

   Here, the polynomial is already in factored form, so we can see that the zeros are:
   $x = 0$ which has multiplicity 2, $x = 1$ which has multiplicity 1, and $x = -3$ which has multiplicity 3.

   (d) $f(x) = x^3(x-2)(x^2-4)^2$

   Here, the polynomial is only partially factored. To put it into fully factored form, notice that $(x^2 - 4) = (x+2)(x-2)$. Then we have $f(x) = x^3(x-2)(x-2)^2(x+2)^2 = x^3(x-2)^3(x+2)^2$
   so we can see that the zeros are:
   $x = 0$ which has multiplicity 3, $x = 2$ which has multiplicity 3, and $x = -2$ which has multiplicity 2.

10. Use the Leading Term Test to determine the end behavior for each polynomial function given.

   (a) $f(x) = x^5 - 4x^3 + 17x^2 + 5x - 14$

   The leading term of this polynomial is x^5. This term has odd degree and positive coefficient.
   Therefore, the left end of the graph falls while the right end rises.

   (b) $f(x) = -3x^8 - 5x^5 + 3x^2 + 17$

   The leading term of this polynomial is $-3x^8$. This term has even degree and negative coefficient.
   Therefore, both ends of the graph fall.

   (c) $f(x) = x^2(2x-1)(x+2)$

   Since this polynomial is in factored form, to find the leading term, we multiply the largest term from each factor together. This gives $x^2(2x)(x) = 2x^4$
   The leading term of this polynomial has even degree and positive coefficient.
   Therefore, both ends of this graph rise.

   (d) $f(x) = -4x^2(x-2)(x^2+1)$

   Since this polynomial is in factored form, to find the leading term, we multiply the largest term from each factor together. This gives $-4x^2(x)(x^2) = -4x^5$
   The leading term of this polynomial has odd degree and negative coefficient.
   Therefore, the left end of the graph rises while the right end falls.

11. Use the Intermediate Value Theorem to show that each polynomial has a zero on the given interval.

   (a) $f(x) = x^4 - 3x^2 + 5x - 7$ on $[1, 2]$

   Notice that $f(1) = 1 - 3 + 5 - 7 = -4$, while $f(2) = (2)^4 - 3(2)^2 + 5(2) - 7 = 16 - 12 + 10 - 7 = 7$
   Since $f(x)$ is continuous, $f(1) < 0$, and $f(2) > 0$, then we must have $f(c) = 0$ for some $1 < c < 2$.

   (b) $f(x) = 4x^3 - 12x^2 + 25x - 100$ on $[3, 5]$

   Notice that $f(3) = 4(27) - 12(9) + 25(3) - 100 = -25$, while $f(5) = 4(125) - 12(25) + 25(5) - 100 = 225$
   Since $f(x)$ is continuous, $f(3) < 0$, and $f(5) > 0$, then we must have $f(c) = 0$ for some $3 < c < 5$.

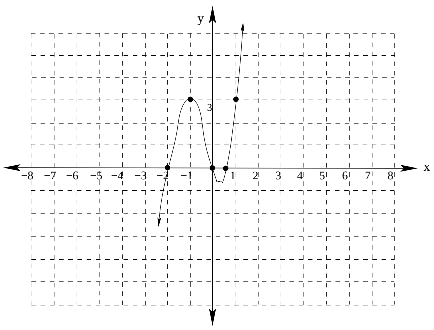
12. Use the Leading Coefficient test to determine the end behavior, find all x-intercepts and their multiplicities, find the y-intercept and any symmetry, and then graph the given polynomial.

   (a) $f(x) = 2x^3 + 3x^2 - 2x$

   The leading coefficient is $2x^3$, which has odd degree and positive coefficient, so the left end falls while the right end rises.

   We find the x-intercepts by solving the related equation $2x^3 + 3x^2 - 2x = 0$ by factoring: $2(2x^2 + 3x - 2) = 0$, or $x(2x - 1)(x + 2) = 0$.
   Then the intercepts are $x = 0$ with multiplicity 1, $x = \frac{1}{2}$ with multiplicity 1, and $x = -2$ with multiplicity 1.

   The y-intercept is $f(0) = 0$, and computing $f(-x)$ shows that this graph is neither even nor odd.
   Also notice that $f(1) = 2 + 3 - 2 = 3$, and $f(-1) = -2 + 3 + 2 = 3$ which gives us a couple more points on the graph.

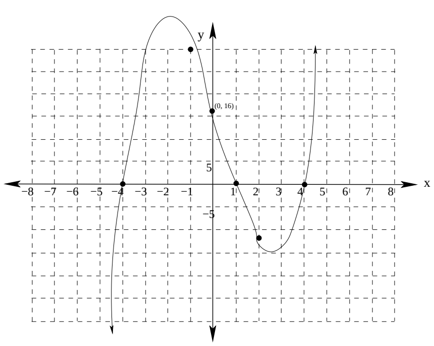


   (b) $f(x) = x^3 - x^2 - 16x + 16$

   The leading coefficient is x^3, which has odd degree and positive coefficient, so the left end falls while the right end rises.

   We find the x-intercepts by solving the related equation $x^3 - x^2 - 16x + 16 = 0$ by factoring: $x^2(x - 1) + -16(x - 1) = 0$, or $(x^2 - 16)(x - 1) = 0$. Then we have $(x + 4)(x - 4)(x - 1) = 0$
   Then the intercepts are $x = -4$ with multiplicity 1, $x = 4$ with multiplicity 1, and $x = 1$ with multiplicity 1.

   The y-intercept is $f(0) = 16$, and computing $f(-x)$ shows that this graph is neither even nor odd.
   Also notice that $f(2) = 8 - 4 - 32 + 16 = -12$, and $f(-1) = -1 - 1 + 16 + 16 = 30$ which gives us a couple more points on the graph.

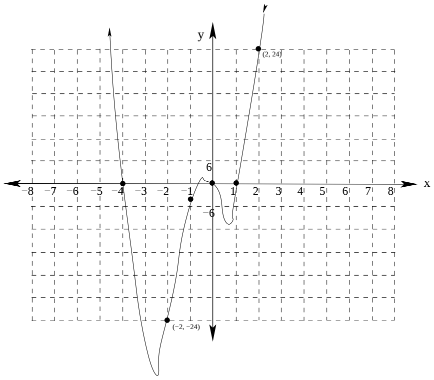


   (c) $f(x) = x^2(x - 1)(x + 4)$

   To find the leading coefficient, we multiply together the largest term from the expansion of each factor: $(x^2)(x)(x) = x^4$, which has even degree and positive coefficient, so both ends are rising.

   The x-intercepts can be read off from the factored form:
   The intercepts are $x = 0$ with multiplicity 2, $x = 1$ with multiplicity 1, and $x = -4$ with multiplicity 1.

   The y-intercept is $f(0) = 0$, and computing $f(-x)$ shows that this graph is neither even nor odd.
   Also notice that $f(2) = 4(1)(6) = 24$, $f(-1) = (1)(-2)(2) = -4$, and $f(-2) = (4)(-3)(2) = -24$ which gives us a few more points on the graph.

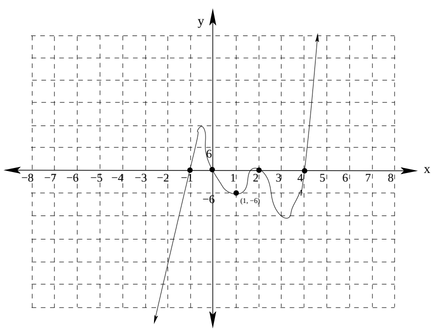


(d) $f(x) = x^3(x+1)(x-2)^2(x-4)$

To find the leading coefficient, we multiply together the largest term from the expansion of each factor: $(x^3)(x)(x^2)x = x^7$, which has odd degree and positive coefficient, so the left end is falling and the right end is rising.

The x-intercepts can be read off from the factored form:
The intercepts are $x = 0$ with multiplicity 3, $x = -1$ with multiplicity 1, $x = 2$ with multiplicity 2, and $x = 4$ with multiplicity 1.

The y-intercept is $f(0) = 0$, and computing $f(-x)$ shows that this graph is neither even nor odd.
Also notice that $f(1) = 1(2)(-1)^2(-3) = -6$ which gives another point on the graph.



13. Use Long Division to find the following Quotients:

(a) $\dfrac{4x^4 - 4x^2 + 6x}{x-4}$

$$
\begin{array}{r|l}
 & 4x^3 + 16x^2 + 60x + 246 \\
\hline
x - 4 & 4x^4 \quad - 4x^2 + 6x \\
 & -4x^4 + 16x^3 \\
 & 16x^3 - 4x^2 \\
 & -16x^3 + 64x^2 \\
 & 60x^2 + 6x \\
 & -60x^2 + 240x \\
 & 246x \\
 & -246x + 984 \\
 & 984
\end{array}
$$

(b) $\dfrac{x^4 - 3x^2 + 7x + 5}{x^2 + 1}$

$$
\begin{array}{r|l}
 & x^2 \quad - 4 \\
\hline
x^2 + 1 & x^4 - 3x^2 + 7x + 5 \\
 & -x^4 - x^2 \\
 & -4x^2 + 7x + 5 \\
 & 4x^2 \quad + 4 \\
 & 7x + 9
\end{array}
$$

(c) $\dfrac{3x^5 - 7x^3 + 5x^2 - 3}{x^2 - x + 3}$

$$\begin{array}{r|llllll}
 & & & 3x^3 & +3x^2 & -13x & -17 \\ \hline
x^2 - x + 3 & 3x^5 & & -7x^3 & +5x^2 & & -3 \\
 & -3x^5 & +3x^4 & -9x^3 & & & \\ \hline
 & & 3x^4 & -16x^3 & +5x^2 & & \\
 & & -3x^4 & +3x^3 & -9x^2 & & \\ \hline
 & & & -13x^3 & -4x^2 & & \\
 & & & 13x^3 & -13x^2 & +39x & \\ \hline
 & & & & -17x^2 & +39x & -3 \\
 & & & & 17x^2 & -17x & +51 \\ \hline
 & & & & & 22x & +48
\end{array}$$

14. Use synthetic division to find the indicated function values:

(a) $f(7)$ if $f(x) = x^3 - 4x^2 + 7x - 5$

$$\begin{array}{r|rrrr}
 & 1 & -4 & 7 & -5 \\
7 & & 7 & 21 & 196 \\ \hline
 & 1 & 3 & 28 & 191
\end{array}$$

The synthetic division given above shows that $f(7) = 191$.

(b) $f(-1)$ if $f(x) = x^5 - 3x^3 + 3x + 4$

$$\begin{array}{r|rrrrrr}
 & 1 & 0 & -3 & 0 & 3 & 4 \\
-1 & & -1 & 1 & 2 & -2 & -1 \\ \hline
 & 1 & -1 & -2 & 2 & 1 & 3
\end{array}$$

The synthetic division given above shows that $f(-1) = 3$.

(c) $f(2)$ if $f(x) = x^7 - 3x^3 + 4x - 1$

$$\begin{array}{r|rrrrrrrr}
 & 1 & 0 & 0 & 0 & -3 & 0 & 4 & -1 \\
2 & & 2 & 4 & 8 & 16 & 26 & 52 & 112 \\ \hline
 & 1 & 2 & 4 & 8 & 13 & 26 & 56 & 111
\end{array}$$

The synthetic division given above shows that $f(2) = 111$.

15. Solve the following equations:

(a) $2x^3 - 3x^2 - 11x + 6 = 0$ given that -2 is a zero of $f(x) = 2x^3 - 3x^2 - 11x + 6$

We begin by using synthetic division to find the remainder when dividing $f(x)$ by $x + 2$:

$$\begin{array}{r|rrrr}
 & 2 & -3 & -11 & 6 \\
-2 & & -4 & 14 & -6 \\ \hline
 & 2 & -7 & 3 & 0
\end{array}$$

The synthetic division given above shows that $f(x) = (x + 2)(2x^2 - 7x + 3)$. This factors further to give $f(x) = (x + 2)(2x - 1)(x - 3)$.

Therefore, the equation $2x^3 - 3x^2 - 11x + 6 = 0$ has solutions $x = -2$, $x = \frac{1}{2}$, and $x = 3$.

(b) $3x^3 + 7x^2 - 22x - 8 = 0$ given that $-\frac{1}{3}$ is a zero of $f(x) = 3x^3 + 7x^2 - 22x - 8$

We begin by using synthetic division to find the remainder when dividing $f(x)$ by $x + 2$:

$$\begin{array}{r|rrrr}
 & 3 & 7 & -22 & -8 \\
-\frac{1}{3} & & -1 & -2 & 8 \\ \hline
 & 3 & 6 & -24 & 0
\end{array}$$

The synthetic division given above shows that $f(x) = (x + \frac{1}{3})(3x^2 + 6x - 24) = (x + \frac{1}{3})(3)(x^2 + 2x - 8)$. This factors further to give $f(x) = (x + \frac{1}{3})(3)(x + 4)(x - 2)$.

Therefore, the equation $3x^3 + 7x^2 - 22x - 8 = 0$ has solutions $x = -\frac{1}{3}$, $x = -4$, and $x = 2$.

16. Use the Rational Zero Theorem to find all possible zeros for each polynomial given.

(a) $f(x) = x^3 + 3x^2 - 6x - 8$

$a_0 = 8$, so possible values for p are: $p = \pm1, \pm2, \pm4, \pm8$

$a_n = 1$ so possible values for q are: $q = \pm1$.

Then list of all possible rational zeros is given by: $\frac{p}{q} = \pm1, \pm2, \pm4, \pm8$

(b) $f(x) = 2x^4 + 3x^3 - 11x^2 - 9x + 15$

$a_0 = 15$, so possible values for p are: $p = \pm1, \pm3, \pm5, \pm15$

$a_n = 2$ so possible values for q are: $q = \pm1, \pm2$.

Then list of all possible rational zeros is given by: $\frac{p}{q} = \pm1, \pm3, \pm5, \pm15, \pm\frac{1}{2}, \pm\frac{3}{2}, \pm\frac{5}{2}, \pm\frac{15}{2}$

(c) $f(x) = 4x^5 - 8x^4 - x + 2$

$a_0 = 2$, so possible values for p are: $p = \pm1, \pm2$

$a_n = 4$ so possible values for q are: $q = \pm1, \pm2, \pm4$.

Then list of all possible rational zeros is given by: $\frac{p}{q} = \pm1, \pm2, \pm\frac{1}{2}, \pm\frac{1}{4}$

17. Find a 5th degree polynomial $f(x)$ for which $\frac{1}{2}$, -1, 2, and $\pm i$ are all zeros and with $f(1) = 12$.

Since $\frac{1}{2}$, -1, 2, and $\pm i$ are all zeros of our polynomial, then $(x - \frac{1}{2})$, $(x + 1)$, $(x - 2)$, $(x - i)$, and $(x + i)$ are all factors of $f(x)$.

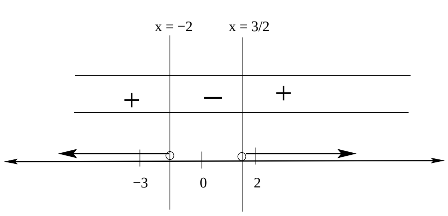
From this, $f(x) = a(x - \frac{1}{2})(x + 1)(x - 2)(x - i)(x + i) = a(x - \frac{1}{2})(x^2 - x - 2)(x^2 + 1)$.
Since $f(1) - 12$, then $a(1 - \frac{1}{2})(1 - 1 - 2)(1 + 1) = 12$, or $a(\frac{1}{2})(-2)(2) = 12$, or $-2a = 12$, so $a = -6$.
Therefore, $f(x) = -6(x - \frac{1}{2})(x^2 - x - 2)(x^2 + 1) = (-6x + 3)(x^4 - x^3 - x^2 - x - 2) = -6x^5 + 9x^4 + 3x^3 - 3x^2 + 9x - 6$

18. Solve the following inequalities. Express your solution in interval notation and graph the solution on a number line.

(a) $2x^2 + x - 6 > 0$

Factoring this gives: $(2x - 3)(x + 2) > 0$, so the boundary values for this inequality are $x = \frac{3}{2}$ and $x = -2$. We use the sign testing method and compute a test value in each region.

If we set $f(x) = 2x^2 + x - 6$, then:
$f(-3) = 18 - 3 - 6 = 9 > 0$
$f(0) = 0 + 0 - 6 = -6 < 0$
$f(2) = 8 + 2 - 6 = 4 > 0$



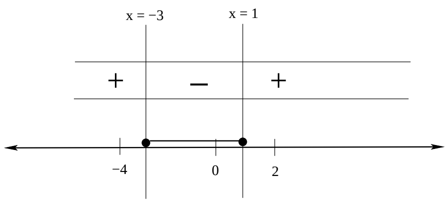
Therefore, the solution is $(-\infty, -2) \cup (\frac{3}{2}, \infty)$

(b) $3x^2 + 5x + 4 \leq 2x^2 + 3x + 7$ (this was a typo on the original handout.)

We begin by moving all of the terms to the left side of the inequality, which gives us: $x^2 + 2x - 3 \leq 0$

Factoring this gives: $(x+3)(x-1) \leq 0$, so the boundary values for this inequality are $x=-3$ and $x=1$. We use the sign testing method and compute a test value in each region.

If we set $f(x) = x^2+2x-3$, then:
$f(-4) = 16-8-3 = 5 > 0$
$f(0) = 0+0-3 = -3 \leq 0$
$f(2) = 4+4-3 = 5 > 0$

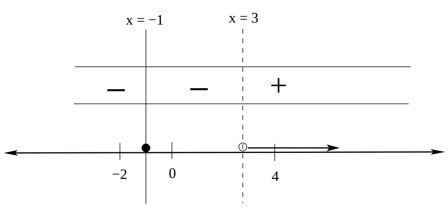


Therefore, the solution is $[-3, 1]$

(c) $\dfrac{x^2+2x+1}{x-3} \geq 0$

Factoring this gives: $\frac{(x+1)^2}{(x-3)} \geq 0$, so the boundary values for this inequality are $x=-1$ and $x=3$. We use the sign testing method and compute a test value in each region.

If we set $f(x) = \dfrac{(x+1)^2}{(x-3)}$, then:
$f(-2) = \frac{(-1)^2}{-5} = -\frac{1}{5} < 0$
$f(0) = \frac{1}{-3} = \frac{-1}{3} < 0$
$f(4) == \frac{25}{2} > 0$

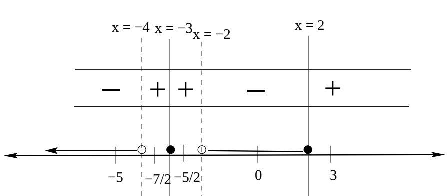


Therefore, the solution is $\{-1\} \cup (3, \infty)$

(d) $\dfrac{(x+3)^2(x-2)}{(x+4)(x+2)} \leq 0$

Since this expression is already factored, we can see that the boundary values for this inequality are $x=-3$, $x=2$, $x=-4$, and $x=-2$. We use the sign testing method and compute a test value in each region.

If we set $f(x) = \dfrac{(x+3)^2(x-2)}{(x+4)(x+2)}$, then:
$f(-5) = \frac{(4)(-7)}{(-1)(-3)} = -\frac{28}{3} < 0$
$f(-\frac{7}{2}) = \frac{(\frac{1}{4})(-\frac{11}{2})}{(\frac{1}{2})(-\frac{3}{2})} > 0$
$f(-\frac{5}{2}) = \frac{(\frac{1}{4})(-\frac{9}{2})}{(\frac{3}{2})(-\frac{1}{2})} > 0$
$f(0) == \frac{(9)(-2)}{(4)(2)} = \frac{-18}{8} < 0$
$f(3) == \frac{(36)(1)}{(7)(5)} = \frac{36}{35} > 0$



Therefore, the solution is $(-\infty, -4) \cup \{-3\} \cup (-2, 2]$

(e) $\dfrac{1}{x-2} \geq \dfrac{3}{x+1}$

We begin by moving all of the terms to the right side of the inequality, which gives us: $0 \geq \frac{3}{x+1} - \frac{1}{x-2}$, or, finding the common denominator, $0 \geq \frac{3(x-2)}{(x+1)(x-2)} - \frac{x+1}{(x+1)(x-2)}$.

Then we have $0 \geq \frac{3x-6-(x+1)}{(x+1)(x-2)}$ or $0 \geq \frac{2x-7}{(x+1)(x-2)}$

Therefore, the boundary values for this inequality are $x = -1$, $x = 2$, and $x = \frac{7}{2}$. We use the sign testing method and compute a test value in each region.

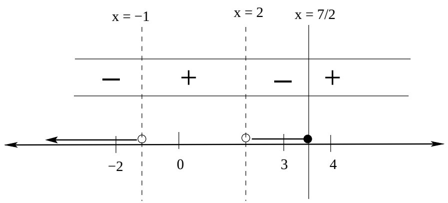
If we set $f(x) = \dfrac{2x-7}{(x+1)(x-2)}$, then:

$f(-2) = \frac{-11}{(-1)(-4)} = -\frac{11}{4} < 0$

$f(0) == \frac{-7}{(1)(-2)} = \frac{7}{2} > 0$

$f(3) = \frac{-1}{(4)(1)} > -\frac{1}{4} < 0$

$f(4) == \frac{1}{(5)(2)} = \frac{1}{10} > 0$



Therefore, the solution is $(-\infty, -1) \cup (2, \frac{7}{2}]$