Math 127 **Name:**____________________________________

Chapter 3 Quiz Solutions

06/16/2011

1. (5 points) Let $f(x) = 2x^2 + 5x - 3$.

   (a) Find the vertex and axis of symmetry.

   We complete the square:

   $f(x) = 2(x^2 + \frac{5}{2}x + ?) - 3 - ?$

   $f(x) = 2\left(x^2 + \frac{5}{2}x + \left(\frac{5}{4}\right)^2\right) - 3 - 2 \cdot \frac{25}{16}$

   $f(x) = 2\left(x + \left(\frac{5}{4}\right)^2\right) - \frac{24}{8} - \cdot\frac{25}{8}$

   $f(x) = 2\left(x + \left(\frac{5}{4}\right)^2\right) - \frac{49}{8}$

   Thus the vertex is $\left(-\frac{5}{4}, -\frac{49}{8}\right)$ and the axis of symmetry is $x = -\frac{5}{4}$

   (b) Find the x-intercepts and y-intercepts of $f(x)$.

   To find the x-intercepts, notice that $2x^2 + 5x - 3 = (2x-1)(x+3)$

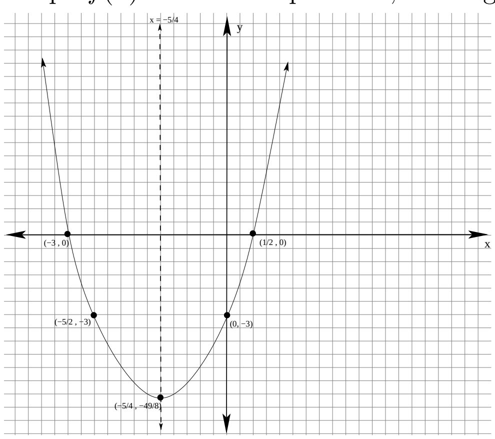
   Thus, the x-intercepts occur when $x = \frac{1}{2}$ and $x = -3$

   Therefore, the x-intercepts are $\left(\frac{1}{2}, 0\right)$ and $(-3, 0)$

   The y-intercept occurs when $f(0) = -3$, or at the point $(0, -3)$

   Notice that, using symmetry, $\left(-\frac{5}{2}, -3\right)$ is another point on the graph.

   (c) Graph $f(x)$ on the axes provided, labeling all key features of the graph.



2. (5 points) Let $f(x) = (x-3)(x-1)^2(x+2)$

   (a) Find the leading term of $f(x)$ and use it to determine the end behavior of the graph of $f(x)$.

   Collecting the highest order of terms from the product above, we see that the leading term is $x \cdot x^2 \cdot x = x^4$

   Therefore, the graph approaches $+\infty$ on both ends.

   (b) Find the zeros of $f(x)$. Also find the multiplicity of each zero and state whether the graph crosses or touches the x-axis at each zero.

   Since the polynomial is in factored form, we can read off the zeros:

   $x = 3$ has multiplicity 1, so the graph crosses the x-axis there.

   $x = 1$ has multiplicity 2, so the graph touches the x-axis and turns around.

   $x = -2$ has multiplicity 1, so the graph crosses the x-axis there.

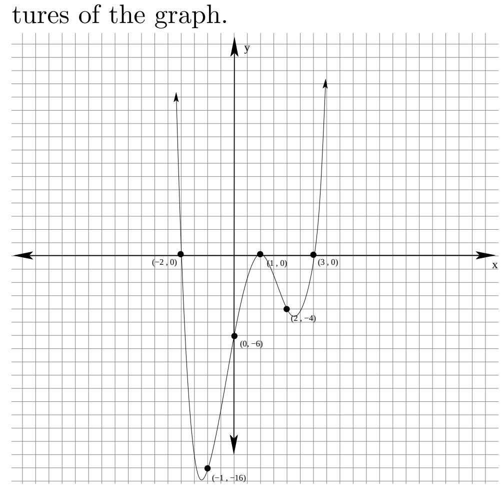
   (c) Find the y-intercept of $f(x)$ and at least one additional point on the graph of $f(x)$.

   The y-intercept occurs at $f(0) = (-3)(-1)^2(2) = -6$

   Other points: $f(-1) = (-4)(-2)^2(1) = -16$

   $f(2) = (-1)(1)^2(4) = -4$

   (d) Graph $f(x)$ on the axes provided, labeling all key features of the graph.



3. (5 points) Let $g(x) = 3x^3 - 2x^2 + 12x - 8$

(a) Use the Rational Root Theorem to list all possible rational zeros for $g(x)$.

$a_0 = -8$, so $p = \pm 1, \pm 2, \pm 4, \pm 8$

$a_n = 3$, so $q = \pm 1, \pm 3$

Then the possible rational roots are: $\frac{p}{q} = \pm 1, \pm 2, \pm 4, \pm 8, \pm\frac{1}{3}, \pm\frac{2}{3}, \pm\frac{4}{3}, \pm\frac{8}{3}$

(b) Use synthetic division to show that $\dfrac{2}{3}$ is a zero of $g(x)$.

	3	−2	12	−8
$\frac{2}{3}$		2	0	8
	3	0	12	0

(c) Find **all** solutions to the equation $g(x) = 0$.

From above, we see that $g(x) = \left(x - \frac{2}{3}\right)(3x^2 + 12) = 3\left(x - \frac{2}{3}\right)(x^2 + 4)$

Then we look at $x^2 + 4 = 0$ to find the remaining zeros. Hence $x^2 = -4$, so $x = \pm\sqrt{-4} = \pm 2i$

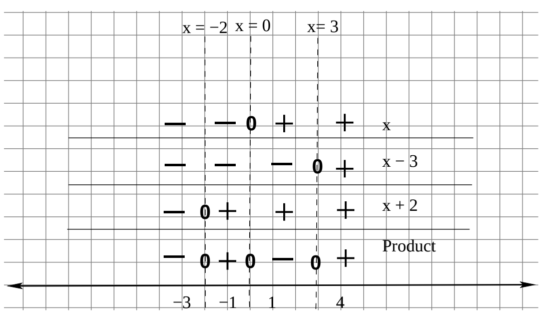
Thus the solutions are $x = \frac{2}{3}$, $x = 2i$, and $x = -2i$.

4. (5 points) Find all solutions to the inequality $x^3 - x^2 \leq 6x$. Graph your solution on a number line. Also give the solution in interval notation.

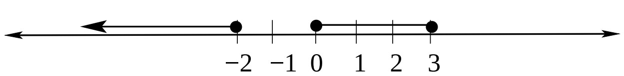
First, we move all terms to the same side: $x^3 - x^2 - 6x \leq 0$.

Next, we factor this expression: $x(x^2 - x - 6) \leq 0$, so $x(x-3)(x+2) \leq 0$

Then the key values for this inequality are $x = 0$, $x = 3$, and $x = -2$. We now make a sign chart:



From this, we see that the solution set is:



Or, in interval notation, $(-\infty, -2] \cup [0, 3]$.