Math 127 - Exam 1 Solutions **Name:**______________________________
Version 1
05/20/2011

You MUST show appropriate work to receive credit

1. (3 points) Express the product $\left(3 \times 10^5\right)\left(5 \times 10^{-3}\right)$ in scientific notation.

   $\left(3 \times 10^5\right)\left(5 \times 10^{-3}\right) = 15 \times 10^{5-3} = 15 \times 10^2 = 1.5 \times 10^3$

2. (3 points) Rewrite the expression $|4 - \sqrt{5}|$ without using the absolute value symbol.

   Since $4 > \sqrt{5}$, $|4 - \sqrt{5}| = 4 - \sqrt{5}$.

3. (3 points each) True or False:

   (a) $(a - b)^2 = a^2 - b^2$

   False.

   Notice that $(a - b)^2 = a^2 - 2ab + b^2$

   (b) $\dfrac{a}{b} + \dfrac{c}{d} = \dfrac{a + c}{b + d}$

   False.

   Notice that $\dfrac{a}{b} + \dfrac{c}{d} = \dfrac{ad}{bd} + \dfrac{cb}{bd} = \dfrac{ad + cb}{bd}$.

   (c) $3^{-2} = -9$

   False.

   Notice that $3^{-2} = \dfrac{1}{3^2} = \dfrac{1}{9}$.

   (d) $\dfrac{a}{b} \div \dfrac{c}{b} = \dfrac{a}{c}$

   True.

   Notice that $\dfrac{a}{b} \div \dfrac{c}{b} = \dfrac{a}{b} \cdot \dfrac{b}{c} = \dfrac{ab}{bc} = \dfrac{a}{c}$.

   (e) $\{b, c, d\} \cup \{d, e, f\} = \{d\}$

   False.

   Since we are finding the union of these sets, $\{b, c, d\} \cup \{d, e, f\} = \{b, c, d, e, f\}$

4. (3 points each) Simplify and/or evaluate each of the following.

   (a) $\dfrac{2(4^2 - 7) - 3\left[7 + (-3)^3\right]}{-3^2 \div (2 - 5)}$

   $= \dfrac{2(16 - 7) - 3\,[7 - 27]}{-9 \div -3} = \dfrac{2(9) - 3\,[-20]}{3} = \dfrac{18 + 60}{3} = \dfrac{78}{3} = 26.$

   (b) $x^2y - 4xy^2 + 4x - 2$ if $x = -2$ and $y = 1$

   $= (-2)^2(1) - 4(-2)(1)^2 + 4(-2) - 2 = 4 + 8 - 8 - 2 = 12 - 10 = 2.$

5. Use properties of exponents and radicals to simplify the following expression. Your answer should have no negative exponents. Assume all variables represent nonnegative numbers.

(a) (5 points) $\left(x^{\frac{1}{3}}\right)^2 \cdot x^3$

$$\left(x^{\frac{1}{3}}\right)^2 \cdot x^3 = x^{\frac{2}{3}} \cdot x^3 = x^{\frac{2}{3}+3} = x^{\frac{2}{3}+\frac{9}{3}} = x^{\frac{11}{3}}.$$

(b) (5 points) $\left(\dfrac{6x^{-3}y^2}{2^{-1}x^2y^{-5}}\right)^3$

$$\left(\frac{6x^{-3}y^2}{2^{-1}x^2y^{-5}}\right)^3 = \left(\frac{6 \cdot 2y^2y^5}{x^2 \cdot x^3}\right)^3 = \left(\frac{12y^7}{x^5}\right)^3 = \frac{12^3y^{21}}{x^{12}} = \frac{1728y^{21}}{x^{15}}.$$

(c) (5 points) $\sqrt[4]{16x^{10}y^{12}}$

$$\sqrt[4]{16x^{10}y^{12}} = \sqrt[4]{2^4x^4x^4x^2y^4y^4y^4} = 2x^2y^3\sqrt[4]{x^2} = 2x^2y^3\sqrt{x}.$$

6. Rationalize all denominators and simplify. Assume all variables represent positive values.

(a) (5 points) $\dfrac{5x}{\sqrt[3]{25x^2y}}$

$$\frac{5x}{\sqrt[3]{25x^2y}} \cdot \frac{\sqrt[3]{5xy^2}}{\sqrt[3]{5xy^2}} = \frac{5x\sqrt[3]{5xy^2}}{\sqrt[3]{125x^3y^3}} = \frac{5x\sqrt[3]{5xy^2}}{5xy} = \frac{\sqrt[3]{5xy^2}}{y}$$

(b) (5 points) $\dfrac{1+\sqrt{x}}{\sqrt{x}-\sqrt{2}}$

$$\frac{1+\sqrt{x}}{\sqrt{x}-\sqrt{2}} \cdot \frac{\sqrt{x}+\sqrt{2}}{\sqrt{x}+\sqrt{2}} = \frac{(\sqrt{1}+\sqrt{x})(\sqrt{x}+\sqrt{2})}{x-2} = \frac{\sqrt{x}+x+\sqrt{2}+\sqrt{2x}}{x-2}.$$

7. (5 points) Simplify the following expression:

$(5x+3y)^2 - (5x-3y)^2$

$(5x+3y)^2 - (5x-3y)^2 = (25x^2 + 15xy + 15xy + 9y^2) - (25x^2 - 15xy - 15xy + 9y^2)$

$= 25x^2 + 30xy + 9y^2 - (25x^2 - 30xy + 9y^2)$

$= 25x^2 + 30xy + 9y^2 - 25x^2 + 30xy - 9y^2$

$= 60xy.$

8. (5 points each) Factor each of the following *completely*. Box your answers.

(a) $6x^2 + 11x + 4$

Using the "ac-split", notice that
$ac = (6)(4) = 24 = (3)(8)$, and $3 + 8 = 11$.
Then $6x^2 + 11x + 4 = 6x^2 + 3x + 8x + 4$
$= 3x(2x + 1) + 4(2x + 1)$
$= (2x + 1)(3x + 4)$.

(b) $x^3 - 4x^2 + 2x - 8$

Using factoring by grouping,
$x^3 - 4x^2 + 2x - 8 = x^2(x - 4) + 2(x - 4)$
$= (x - 4)(x^2 + 2)$.

(c) $x^4 - 16$

Notice that this is a difference of squares,
so $x^4 - 16 = (x^2 + 4)(x^2 - 4)$.

The second term is also a difference of squares,
so this factors further as $(x^2 + 4)(x + 2)(x - 2)$.

(d) $27x^3 - 8$

Notice that this is a difference of cubes: $((3x)^3 - (2)^3)$. Therefore, using the special formula for a difference of cubes:

$27x^3 - 8 = (3x - 2)(9x^2 + 6x + 4)$

9. (7 points each) Perform the operations indicated and simplify each of the following as much as possible. Your answer should be completely reduced and should contain no complex fractions.

(a) $\dfrac{x^2 + 4x - 5}{x^2 + 5x + 6} \div \dfrac{x^2 - x}{x^2 + 8x + 15}$

Factoring, we have $\dfrac{(x + 5)(x - 1)}{(x + 3)(x + 2)} \div \dfrac{x(x - 1)}{(x + 5)(x + 3)}$

Next, we change from multiplication to division: $\dfrac{(x + 5)(x - 1)}{(x + 3)(x + 2)} \cdot \dfrac{(x + 5)(x + 3)}{x(x - 1)}$

Now, we divide out any common factors: $\dfrac{x + 5}{(x + 2)} \cdot \dfrac{(x + 5)}{x}$.

Thus our final simplified answer is: $\dfrac{(x + 5)^2}{x(x + 2)}$

(b) $\dfrac{11x - 1}{x^2 - 2x - 3} + \dfrac{3x}{x + 1}$

First, we factor in order to obtain: $\dfrac{11x - 1}{(x - 3)(x + 1)} + \dfrac{3x}{x + 1}$. Notice that the LCD is: $(x - 3)(x + 1)$

Multiplying to get all terms over the LCD gives:

$$\frac{11x - 1}{(x - 3)(x + 1)} + \frac{3x}{x + 1} \cdot \frac{(x - 3)}{(x - 3)} = \frac{11x - 1 + 3x(x - 3)}{(x - 3)(x + 1)}.$$

Simplifying, we get:

$$\frac{11x - 1 + 3x^2 - 9x}{(x - 3)(x + 1)} = \frac{3x^2 + 2x - 1}{(x - 3)(x + 1)}$$

$$= \frac{(3x - 1)(x + 1)}{(x - 3)(x + 1)} = \frac{3x - 1}{x - 3}.$$

(c) $\dfrac{\frac{1}{x}+\frac{1}{3x}}{\frac{4}{x}-\frac{5}{6x}}$

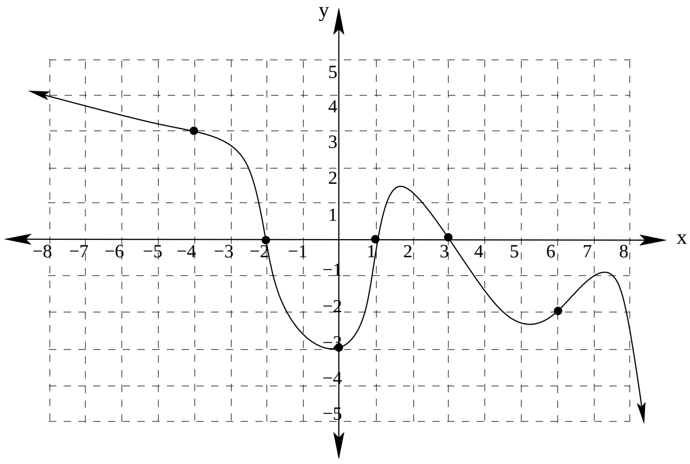
We begin by finding common denominators and combining the fractions separately:

$$\frac{\frac{1}{x}+\frac{1}{3x}}{\frac{4}{x}-\frac{5}{6x}} = \frac{\frac{3}{3x}+\frac{1}{3x}}{\frac{24}{6x}-\frac{5}{6x}} = \frac{\frac{4}{3x}}{\frac{19}{6x}}.$$

Next, we rewrite division as multiplication by the reciprocal:

$$\frac{\frac{4}{3x}}{\frac{19}{6x}} = \frac{4}{3x}\cdot\frac{6x}{19} = \frac{4\cdot 6x}{3x\cdot 19} = \frac{8}{19}.$$

10. (2 points each) Answer each of the following questions based on the graph shown below:



(a) List the coordinates of the x-intercepts of this graph.

$(-2, 0)$, $(1, 0)$, and $(3, 0)$

(b) List the coordinates of the y-intercepts of this graph.

$(0, -3)$

(c) Give the x coordinate of the point(s) where $y = 3$

$x = -4$

(d) Give the y coordinate of the point(s) where $x = 6$

$y = -2$