Math 127 - Exam 2 **Name:**______________________________
Version 1
05/27/2011

You MUST show appropriate work to receive credit

1. (3 points each) True or False: (Be sure to write your answer to each part legibly)

   (a) If $x(x+3) = 5$, then $x = 5$, or $x = 2$.

   False. The difficulty here is that since the right hand side of this equation is not zero, we cannot use the zero factor property. If we try to check these solutions, notice that $5(5+3) = 5(8) = 40$ and $2(2+3) = 2(5) = 10$, so neither of these values are in the solution set of this equation.

   (b) Every equation of the form $ax^2 + bx + c = 0$ has two real solutions.

   False. There are two things that can go wrong here. If $b^2 - 4ac = 0$, then this equation will have only one solution. On the other hand, if $b^2 - 4ac < 0$, then this equation will have two *non*-real solutions.

2. (6 points) Use completing the square to solve the quadratic equation $3x^2 - 3x - 4 = 0$.

$$3x^2 - 3x = 4$$
$$x^2 - x = \tfrac{4}{3}$$
$$x^2 - x + \left(\tfrac{1}{2}\right)^2 = \tfrac{4}{3} + \left(\tfrac{1}{2}\right)^2$$
$$(x - \tfrac{1}{2})^2 = \tfrac{4}{3} + \tfrac{1}{4}$$
$$(x - \tfrac{1}{2})^2 = \tfrac{16}{12} + \tfrac{3}{12}$$
$$(x - \tfrac{1}{2})^2 = \tfrac{19}{12}$$
$$x - \frac{1}{2} = \pm\sqrt{\frac{19}{4\cdot 3}}$$
$$x = \frac{1}{2} \pm \frac{\sqrt{19}}{2\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}}$$
$$x = \frac{1}{2} \pm \frac{\sqrt{57}}{6}$$

3. Perform the indicated operations and express your answer in the form $a + bi$:

   (a) (4 points) $(3i - 2)^2$

   $(3i-2)^2 = (3i-2)(3i-2) = 9i^2 - 6i - 6i + 4 = -9 - 12i + 4 = -5 - 12i$

   (b) (4 points) $\dfrac{5+4i}{2-3i}$

$$\frac{5+4i}{2-3i} \cdot \frac{2+3i}{2+3i} = \frac{10+8i+15i+12i^2}{4+6i-6i-9i^2} = \frac{10-12+23i}{4+9} = \frac{-2+23i}{13} = -\frac{2}{13} + \frac{23}{13}i$$

   (c) (2 points) i^{161}

   Recall that $i^4 = 1$. Then, $i^{161} = \left(i^4\right)^{40} \cdot i = 1 \cdot i = i = 0 + 1i$.

4. (6 points each) Find all solutions to the following equations:

(a) $\dfrac{7}{10} - \dfrac{x}{3} = \dfrac{2x-5}{2}$

We multiply by the LCD, 30: $30 \cdot \left[\dfrac{7}{10} - \dfrac{x}{3} = \dfrac{2x-5}{2}\right]$

$7(3) - 10x = 15(2x-5)$, or $21 - 10x = 30x - 75$.

Then $96 = 40x$, so $x = \frac{96}{40} = \frac{12}{5}$.

(b) $\dfrac{3x-2}{6x+1} = \dfrac{2x+5}{4x-3}$

We multiply by the LCD, $(6x+1)(4x-3)$: $[(6x+1)(4x-3)]\dfrac{3x-2}{6x+1} = [(6x+1)(4x-3)]\dfrac{2x+5}{4x-3}$

$(4x-3)(3x-2) = (6x+1)(2x+5)$, or $12x^2 - 8x - 9x + 6 = 12x^2 + 30x + 2x + 5$

then $12x^2 - 17x + 6 = 12x^2 + 32x + 5$, so $1 = 49x$, thus $x = \frac{1}{49}$.

(c) $3x^2 - 2x = -1$

We immediately rearrange the terms to put this quadratic into standard form: $3x^2 - 2x + 1 = 0$
Since this quadratic equation does not factor, the most straightforward way to solve this is to use the quadratic formula.
Notice that $a = 3$, $b = -2$, and $c = 1$.
Then $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \dfrac{2 \pm \sqrt{4 - 4(3)(1)}}{2(3)} = \dfrac{2 \pm \sqrt{4-12}}{6}$
So $x = \dfrac{2 \pm \sqrt{-8}}{6} = \dfrac{2 \pm 2i\sqrt{2}}{3} = \dfrac{1 \pm i\sqrt{2}}{3}$

(d) $2x^3 - 3x^2 + 8x = 12$

Notice that this polynomial factors by grouping: $x^2(2x-3) + 4(2x-3) = 0$, or $(2x-3)(x^2+4) = 0$

Using the zero product property to split into 2 cases, we see that either $2x - 3 = 0$, so $x = \frac{3}{2}$
or $x^2 + 4 = 0$, in which case, $x^2 = -4$, so $x = \pm\sqrt{-4} = \pm 2i$.
Then $x = \frac{3}{2}$ or $x = 2i$ or $x = -2i$.

(e) $\sqrt{x+8} - \sqrt{x-4} = 2$

This problem ends up being a bit nicer if we rearrange: $\sqrt{x+8} = 2 + \sqrt{x-4}$
Squaring both sides: $\left(\sqrt{x+8}\right)^2 = \left(2+\sqrt{x-4}\right)^2$
which gives $x + 8 = 4 + 4\sqrt{x-4} + x - 4$
Moving terms to isolate the remaining radical gives: $8 = 4\sqrt{x-4}$, or $2 = \sqrt{x-4}$
Squaring again gives: $4 = x - 4$, or $x = 8$
It is imperative that we check our answer, since the method of squaring can introduce extraneous solutions.
If $x = 8$, then $\sqrt{x+8} - \sqrt{x-4} = \sqrt{16} - \sqrt{4} = 4 - 2 = 2$, so this solution does check.
Hence this equation has one solution, $x = 8$.

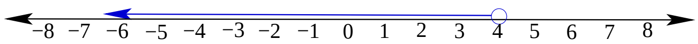
(f) $t^4 - 9t^2 + 20 = 0$

Here, we can either factor this expression, or, to make it a bit easier, we can substitute using $u = t^2$ and $u^2 = t^4$.
This gives $u^2 - 9u + 20 = 0$, or $(u-5)(u-4) = 0$
Therefore, either $u = 5$ or $u = 4$
That is, either $t^2 = 5$ or $t^2 = 4$
But then either $t = \pm\sqrt{5}$ or $t = \pm\sqrt{4} = \pm 2$.
Notice that all 4 of these solutions check.

5. Solve the following inequalities. Graph your solution on a number line, and express your answer in interval notation.

(a) (5 points) $8x - (3x - 2) < 2(x + 7)$

Simplifying, we get $8x - 3x + 2 < 2x + 14$, or $5x + 2 < 2x + 14$
Moving terms to isolate x, we get $3x < 12$, or $x < 4$.
In interval notation, this is: $(-\infty, 4)$

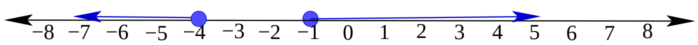


(b) (6 points) $|2x + 5| - 2 \geq 1$

We first isolate the absolute value part of the inequality: $|2x + 5| \geq 3$
Since the constant term is positive, this inequality has solutions. We proceed by splitting into two cases.

Positive case: $2x + 5 \geq 3$	Negative case: $-(2x+5) \geq 3$ or $2x + 5 \leq -3$
Then $2x \geq -2$	or $2x \leq -8$
Thus $x \geq -1$	or $x \leq -4$

Which, in interval notation gives: $(-\infty, -4] \cup [-1, \infty)$



6. (6 points) Fred and Wilma start from the same point in Bedrock and travel on a straight road. Fred travels at 30 mph, while Wilma travels at 50 mph. If Wilma starts 3 hours after Fred, find the distance they travel before Wilma catches up to Fred. [You must use algebra and the complete problem solving process to get full credit on this problem.]

Since we are taking about simple motion, the basic equation is $d = rt$. We will start by setting up separate models for Fred and Wilma and then we will compare the two. Our main variables will be t, the total time Fred spent traveling, and d, the distance both Fred and Wilma travel.

For Fred, $d_1 = r_1t_1$, where $r_1 = 30$ miles per hour, and $t_1 = t$, the total travel time for Fred.

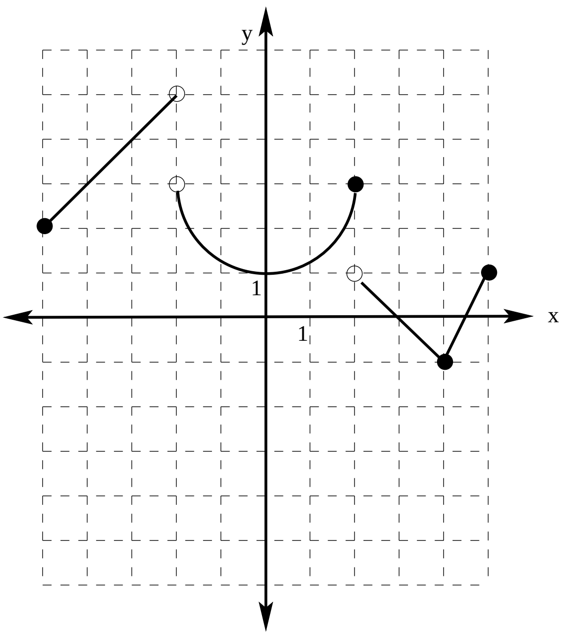
For Wilma, $d_2 = r_2t_2$, where $r_1 = 50$ miles per hour, and $t_2 = t - 3$ (since Wilma started out 3 hours after Fred, her total travel time will end up being 3 hours less than Fred's).

Notice that since we are interested in the time when Wilma caught up to Fred, $d_1 = d_2 = d$.

Then we have the equation $r_1t_1 = r_2t_2$, or $30t = 50(t - 3)$.
Therefore, $30t = 50t - 150$, or $150 = 20t$.
Then $t = \frac{150}{20} = 7.5$ hours. That is, it will take Wilma 7.5 hours to catch up with Fred.

Therefore, Fred and Wilma will have traveled $30(7.5) = 225$ miles ($50(4.5) = 225$ miles) when Wilma catches up with Fred.

7. For the given graph of $f(x)$, find the following:



(a) (2 points) $f(3)$

$f(3) = 0$

(b) (2 points) x, when $f(x) = 1$

$x = 0$ and $x = 5$.

(c) (3 points) The domain of f

Domain: $[-5, -2) \cup (-2, 5]$.

(d) (3 points) The range of f

Range: $[-1, 5)$.

(e) (3 points) The interval(s) where f is decreasing.

$(-2, 0)$ and $(2, 4)$.

8. Let $f(x) = x^2 - x$. Find and simplify the following:

(a) (2 points) $f(3)$

$f(3) = 3^2 - 3 = 9 - 3 = 6$

(b) (3 points) $f(a+1)$

$f(a+1) = (a+1)^2 - (a+1) = a^2 + 2a + 1 - a - 1 = a^2 + a = a(a+1)$.

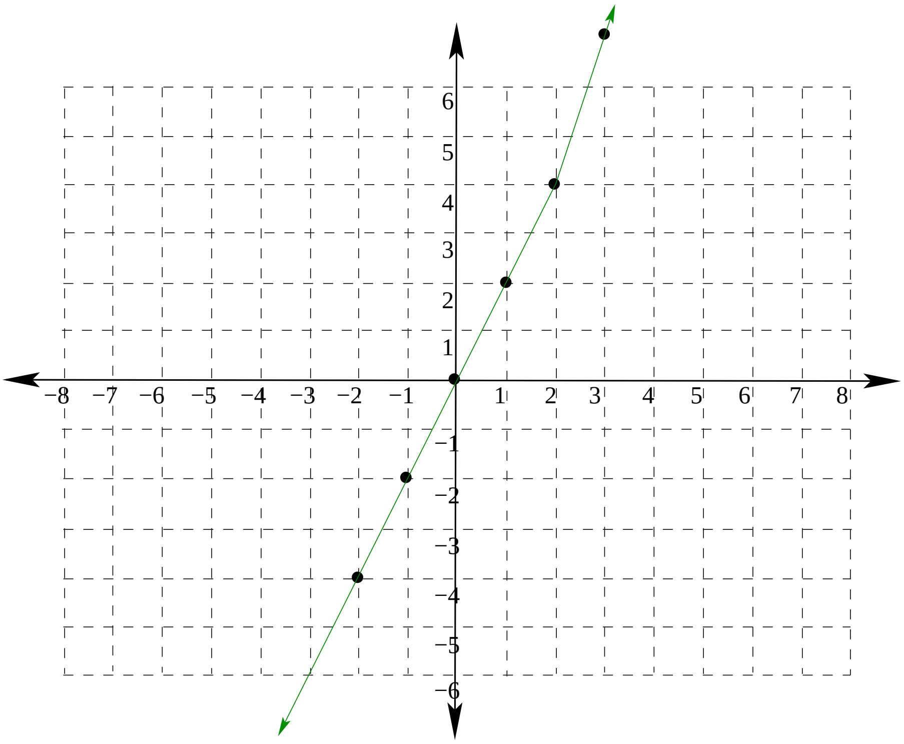
(c) (4 points) $\dfrac{f(a+h) - f(a)}{h}$

Notice that $f(a) = a^2 - a$, and $f(a+h) = (a+h)^2 - (a+h) = a^2 + 2ah + h^2 - a - h$.

Therefore, $\dfrac{f(a+h) - f(a)}{h} = \dfrac{a^2 + 2ah + h^2 - a - h - a^2 + a}{h} = \dfrac{2ah + h^2 - h}{h} = 2a + h - 1$

9. (5 points) Draw the graph of the function $f(x) = \begin{cases} 2x & \text{if } x < 2 \\ 3x - 2 & \text{if } x \geq 2 \end{cases}$.

Label your axes and at least 4 points on the graph.



x	y
-2	-4
-1	-2
0	0
1	2
2	4
3	7
4	10

Extra Credit: (5 points) Use completing the square on the general quadratic polynomial $ax^2 + bx + c = 0$ to derive the quadratic formula

$ax^2 + bx + c = 0$

$ax^2 + bx = -c$

$x^2 + \frac{b}{a}x = -\frac{c}{a}$

$x^2 + \frac{b}{a}x + \left(\frac{b}{2a}\right)^2 = -\frac{c}{a} + \left(\frac{b}{2a}\right)^2$

$(x + \frac{b}{2a})^2 = -\frac{c}{a} + \frac{b^2}{4a^2}$

$x + \frac{b}{2a} = \pm\sqrt{-\frac{c}{a} + \frac{b^2}{4a^2}} = \pm\sqrt{\frac{-4ac}{4a^2} + \frac{b^2}{4a^2}} = \pm\sqrt{\frac{-4ac + b^2}{4a^2}}$

$x = -\frac{b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$